SUSTAINABLE DYNAMISM: A Regional Economic Development Strategy of Continuous Reinvention

Volume II: Background and Formal Framework

Sustainable Dynamism

A new approach to economic development, and its potential to generate continuous growth in regional per capita income and GDP, is predicated on a regional economy's ability to exploit successive waves of new technologies and innovations by fostering an economic environment conducive to entrepreneurial activity and new firm formation that will produce a sustainable process whereby new products and services are continually introduced into the market.

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SUSTAINABLE DYNAMISM:
A Regional Economic Development Strategy of Continuous Reinvention

VOLUME II: Background and Formal Framework

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Sustainable Dynamism: A Regional Economic Development Strategy of Continuous Reinvention (Volume II)

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FOREWORD

This is Volume II of a two-volume research report on the implementation of the recommendations of ***Benchmarking Growth in Demand-Driven Labor Markets.***[1] Volume I presents a general framework for implementing a strategy of fostering an economic environment conducive to entrepreneurial activity and new-firm formation that will produce a sustainable process, whereby new products and services are continually introduced into the market. In addition, the critical role of workforce development policies and programs are addressed. To that end, the first volume lays out specific programs and strategies, and puts them within the context of recent work done on technology transfer and Connecticut's future economic prospects. It lays out a formal context for constructing a framework for a growth and development strategy. It then provides an operational definition of *sustainable dynamism*, which is grounded in the idea that such a set of economic conditions would characterize a region where innovation itself is its "leading product." The birth and evolution of four science cities suggests a framework, within a workforce-investment context, for implementing a set of policies that would put Connecticut's regional economies on track to achieving sustainable dynamism.

This second volume lays out the background and development of the formal framework that provides the context for the implementation strategies presented in Volume I. The focus in Volume II is on the development of economic growth theory, the current emphasis on technological change as endogenous to the growth and development process, and the implications for the programs and policies recommended in Volume I. For those who would prefer a more formalized development of the ideas that guide the strategies presented in Volume I, this volume (Volume II) should provide that framework, as well providing references, and a springboard for, further research on regional growth and development.

[1] McPherron, Patrick, "Benchmarking Growth In Demand-Driven Labor Markets – 2006" OCCASIONAL PAPER (December 2006) Office of Research, Connecticut Department of Labor: Wethersfield, CT.

EXECUTIVE SUMMARY

Volume II details the formal framework that provides the context for the discussion on implementing the policies and programs in Volume I, which address the issues and challenges identified in *Benchmarking Growth in Demand-Driven Labor Markets*. It elaborates on, and extends, the Audretsch, Keilbach, and Lehmann (2006) approach to the development of economic growth theory after World War II. They partition the Post-WW II Era into three historical periods (1.) Technology as Exogenous: the Solow Model, (2.) Technology as Endogenous: the Romer-Lucas Model, and (3.) the Entrepreneurial Economy. Central to their approach, and that followed in Volume I, is the treatment of technology and knowledge by each of the three growth paradigms, and their implications about the changing role of the entrepreneur and the exploitation of knowledge for the development and introduction of new products and services into the market.

The Keynesian/Post-Keynesian oriented Harrod-Domar and circular-cumulative causation economic growth models, and the Neoclassical/Endogenous economic growth models presented in this volume were developed to explain economic growth at the national level. Later, these models were adapted to explain the dynamics of regional growth and development. The first economic model developed specifically to study local and regional economic growth was the *Economic Base Model*, or *Economic Base Analysis*, developed by Hoyt[2] and Wiemer (1939)[3] in order to estimate the prospects of local economies. It pre-dated Neoclassical growth theory, and coincided with the publication of Domar's (1939) paper. Later, in 1962, Tiebout forged the connections between economic base theory and Keynesian theory.[4] The key driver of growth in the economic-base model is trade. It focuses on macroeconomic analysis, as opposed to microeconomic analysis, and the basic level of aggregation is the city or region. The critical link is between two broad, aggregate industry groupings: the *basic sector* and the *non-basic sector*. The region's growth in per capita income and GDP is dependent on the income earned from, and brought into the region by, the basic sector through selling its goods and services outside the region. Thus, the health of the non-basic sector is dependent on the performance of the basic sector.

For the last half of the 20th Century, economic base theory was, and it continues to be into the 21st Century, the most widely used model for analyzing economic growth at the local and regional level. However, also in the last half of the 20th Century, both the Harrod-Domar and Neoclassical models were adapted to explain regional growth. In 1964, Borts and Stien first adapted the Neoclassical model to explain regional growth. And, in 1978, Ghali, Akiyama, and Fujiwara used the Neoclassical framework to study economic growth in the U.S. states. Subsequently, it has been applied to study economic growth at the regional level, not only in the U.S., but also for analyzing regional economic growth in other areas of the world, particularly the regional economies of the European Union. In 1969, Richardson adapted the Harrod-Domer model to explain economic dynamics at the regional level.

In tracing the development of economic growth theory, from its inception to its application to regional growth and development, two major approaches to economic growth are presented: *Neoclassical growth theory* and its extensions, and *Keynesian-based growth theory*. Very simply, Neoclassical growth theory can be thought of as focusing on problems of supply, while assuming sufficient demand. Keynesian-based growth theories can be thought of as focusing on deficiencies in demand that constrain production (output) and growth. The Harrod-Domar model, a Keynesian-oriented model, focuses on the interaction between supply *and* demand, and how the interplay and feedbacks between the two drive growth and fluctuations in the economy.[5] Another set of Keynesian-based growth and development models, circular-

[2] Homer Hoyt was a planner at the U.S. Federal Housing Administration.
[3] Mazilia and Feser (1999, 2000) f. 1, p. 76 and Capello (2007) p.110. Although, according to the entry in Wikipedia, it was developed by Robert Murray Haig in his work on the Regional Plan of New York in 1928.
[4] ibid, p. 78.
[5] In fact, the Harrod-Domar Model was originally developed to explain the Business Cycle.

cumulative causation, developed by Myrdal and formalized by Kaldor, also focuses on the interaction between supply and demand, and how this interaction ignites a chain-reaction process that generates a virtuous circle of cumulative causation propelling a region toward a trajectory of growth and development.

The first step to developing a framework is to retrace developments in growth theory in the Post-World War II (WW II) Era, with an emphasis on the roles that knowledge and technology have played in the formulation in each of the growth paradigms. Following Audretsch, Keibach, and Lehmann (2006), the development of economic growth theory in the Post-WW II Era, is partitioned into three historical periods: (1.) Technology as Exogenous: the Solow Model; (2.) Technology as Endogenous: the Romer-Lucas Model; and (3.) the Entrepreneurial Economy. However, before tracing the development of growth theory through this framework, it is important to first introduce the critical concept of the *production function*.

From Robert Solow's 1956 paper on, Neoclassical growth theory has been based on the production function. Thus, a basic grasp of this production relationship is essential for understanding the subsequent discussion of the evolution of economic growth theory since World War II. The production function relates the quantities of capital (K) and labor (L) inputs used to produce a given level of output (Q). The algebraic expression for the production relationship is stated as:

$$Q = Q(K,L)$$

Where: Q = Quantity of output produced
K = Amount of the capital input used to produce Q
L = Amount of the labor input used to produce Q
Q(.) = Indicates a functional relationship between the inputs and the resulting output produced.

The above production relationship implies that the minimum combination of inputs of K and L were used to produce some given level of output, Q.

One of the most well known concepts from microeconomics is the *Law of Diminishing Returns*, or *Variable Proportions*. It has important implications for the focus of both this volume (Volume II) and Volume I of this report, on the evolution of, and the competing theories of, economic growth and development. It applies to the short-run perspective of production, where one factor input (in the above expression for the production function), usually capital (K), is held constant. As a variable input, such as labor (L), is added to a set of fixed inputs, such as a given size plant, and holding technology constant, output increases rapidly. That is, output, or Q, increases at an increasing rate. Using an auto plant as an example, as more and more workers are added to the same assembly line of a given auto plant, with a given set of technologies, for a given, short-run time period, to produce lot-runs of cars, the number of cars produced would begin to level off and grow at a constant rate, and then grow at a decreasing rate. This is because, at some point, additional workers would become redundant. Then, output, or Q, would be increasing at a decreasing rate. There would be too many workers on the line trying to produce cars in a fixed-sized plant. In general, when more and more of a variable input is added to a fixed input (with technology held constant), for a given time period, output, at first, increases at an increasing rate, then it increases at a constant rate, and eventually at a decreasing rate. If the process is carried far enough, and the variable input reaches the saturation point, relative to the fixed input, then there would actually be negative returns to scale. From Robert Solow's 1956 paper on, Neoclassical growth theory has been based on the production function. Thus, the *law of diminishing returns* is a critical concept for a discussion on the Post-World War II developments in Neoclassical growth theory, and its extensions, as they are framed within the context of the production function and its implications.

Before introducing Neoclassical growth theory, it is essential to introduce its antecedent, and the motivation for, its development. The Harrod-Domar model was initially created to help analyze the business cycle; however, it was later adapted to explain economic growth. Its implications were that growth depends on the quantity of labor and capital, and that more investment leads to capital accumulation, which generates economic growth. The Harrod-Domar model predicts that if it is expected that output will grow, investment will increase to meet the extra demand. The problem arises when actual growth either exceeds or falls short of warranted growth expectations. A vicious cycle can be created where the difference is exaggerated by attempts to meet the actual demand, causing economic instability. Thus, to be stable, growth must follow a "razor's edge" path.

The first period of the Post-WW II Era can be characterized as the "Solow Economy." Robert Solow's 1956 article was largely addressed to the pessimism about the *razor's-edge* path that the economy must maintain to sustain full-employment growth, which is built into the Harrod-Domar growth model. Solow's work changed the approach that economists took to study growth. From then on, the production function model has been the basis for explaining the determinants of economic growth. The production-function approach relates measures representing the two fundamental factors of production introduced in the expression above: physical capital (K) and unskilled labor (L). These two fundamental factor inputs were used as the basis for explaining variations in growth rates over time in a single country, or across countries in a cross-sectional context. The *unexplained residual*, which typically accounted for a large share of the unexplained variance in growth rates, was attributed to *technological change***.** Solow acknowledged that technical change contributed to economic growth, but in terms of the formal model, it was considered to be "manna from heaven." It took place outside the Neoclassical framework; that is, it was not explained within the model, and it could only be introduced by showing upward shifts in the aggregate production function. First proposed by Romer (1986, 1987), *endogenous growth theory* maintained the orthodox Neoclassical growth-accounting framework, but dispensed with the need for an exogenous technology residual. Unlike Romer's focus on firm-specific capital, Lucas's version of the endogenous growth model is based on the *level of human capital*. According to the Lucas model, the portion of output attributed to the technology residual in the Neoclassical growth model should actually be attributed entirely to labor through human-capital acquisition. A fundamental implication emerging from the models of endogenous growth was that higher economic growth rates could be obtained through knowledge investments.

Critical to both the Romer and Lucas models, is the internalizing of technological progress within the Neoclassical production-function framework by introducing knowledge as an explicit factor of production. As discussed above, in the Solow growth model, technology was exogenous, and thus resulted in an upward shift in the aggregate production function. In contrast, endogenous growth theory sought to identify the mechanism that explained technological progress over time, and to show that it was a product of the internal processes of the economy.

In their *theory of endogenous entrepreneurship*, Audretsch, Keilbach, and Lehmann (2006) emphasize the critical delineation between *information* and *knowledge*. While advances in information technology have rendered the cost of transmitting information across space trivial, the cost of transferring knowledge across space still increases rapidly with distance. In addition to the distinction between information and knowledge, also critical to the development of a model of entrepreneurship and growth, is the idea that there is a barrier to translating new knowledge into new *economic* knowledge. Audretsch, et al (2006) formalize this idea in their concept of the *knowledge filter*. Further, not only is the knowledge filter the consequence of the basic conditions inherent in new knowledge, but it is also what creates the opportunity for entrepreneurship in the *knowledge spillover theory of entrepreneurship.*

Their observation that knowledge conditions dictate the relative advantages in exploiting opportunities arising from investments in knowledge of incumbents versus small and large enterprises is predicated on the distinction between two knowledge regimes: the *routinized technological regime* versus the *entrepreneurial technological regime*. The routinized technological regime reflects knowledge conditions where the large incumbent firms have the innovative advantage. Conversely, in the entrepreneurial technological regime, knowledge conditions give the advantage to small firms. In their formal model of endogenous entrepreneurship, Audretsch, et al (2006) emphasize, not only that the capacity of each regional economy to generate entrepreneurial spillovers and commercialize knowledge is not the same, but, in addition, just as the knowledge filter should not be assumed to be impermeable, the capacity of a region's economy to generate knowledge spillovers via entrepreneurship to permeate the knowledge filter should not be assumed to be automatic. Consequently, Audretsch, et al (2006) designate the remaining *untapped* part as *opportunities* that can be taken on by new firms. They denote this part as *entrepreneurial opportunities*, and it is explicitly expressed as a term in their specification of the production function.

Based on the above model of the knowledge spillover theory of entrepreneurship and economic growth, as well as their framework for analyzing the recognition of and then acting upon entrepreneurial opportunities, Audretsch, et al derive the following hypotheses concerning the determinants of entrepreneurship and its Impact on economic performance:

- *Endogenous Entrepreneurship Hypothesis*: Entrepreneurship will be greater in the presence of higher investments in new knowledge, ceteris paribus.

- *Economic Performance Hypothesis*: Entrepreneurial activity will increase the level of economic output since entrepreneurship serves as a mechanism facilitating the spillover and commercialization of knowledge.

- *Location Hypothesis:* Knowledge spillover entrepreneurship will tend to be spatially located within close geographic proximity to the source of knowledge actually producing that knowledge.

- *Entrepreneurial Performance Hypothesis*: Opportunities for knowledge-based entrepreneurship, and therefore performance of knowledge-based start-ups, is superior when they are able to access knowledge spillovers through geographic proximity to knowledge sources, such as universities, when compared to their counterparts without a close geographic proximity to a knowledge source.

- *Entrepreneurial Access Hypothesis*: Knowledge-based entrepreneurial firms will strategically adjust the composition of their boards and ma3nagers toward higher levels of knowledge and human capital so they can contribute to the access and absorption of external knowledge spillovers.

- *Entrepreneurial Finance Hypothesis*: Knowledge-based entrepreneurial firms will tend to be financed from equity-based sources, such as venture capital, and less typically from traditional debt-based sources, such as banks.

After introducing their hypotheses in Chapter 4, Audretsch, et al report the results of their empirical tests of those hypotheses in the subsequent chapters of *Entrepreneurship and Economic Growth* (2006). Their findings are an important part of the argument for linking an entrepreneurship program to economic and workforce development programs and policies presented in this report.

I. INTRODUCTION AND PLAN OF APPROACH

This volume develops the formal framework that provides the context for the discussion on implementing the policies and programs in Volume I, which address the issues and challenges identified in *Benchmarking Growth in Demand-Driven Labor Markets*. What follows elaborates on, and extends, the Audretsch, Keilbach, and Lehmann (2006) approach to the development of economic growth theory after World War II, presented in Section III, "A Formal Context for a Growth and Development Strategy," in Volume I of this report.

The Neoclassical/Endogenous and Keynesian/Post-Keynesian economic growth models presented in this volume were developed to explain economic growth at the national level. Subsequently, they were adapted to explain regional growth. The first economic model developed specifically to study local and regional economic growth was the *Economic Base Model*, or *Economic Base Analysis*, developed by Hoyt[6] and Wiemer (1939)[7] in order to estimate the prospects of local economies. It pre-dated Neoclassical growth theory, and coincided with the publication of Domar's (1939) paper (see Section III, this volume). Later, in 1962, Tiebout forged the connections between economic base theory and Keynesian theory.[8] The key driver of growth in the economic base model is trade. It focuses on macroeconomic analysis, as opposed to microeconomic analysis, and the basic level of aggregation is the city or region. The critical link is between two broad, aggregate industry groupings: the *basic sector* and the *non-basic sector*.

The basic sector is that broad grouping of industries that export their goods and services outside the city or region to other regions or internationally, or both. Thus, for this group of industries, their market lies outside the region. The non-basic sector is composed of the broad grouping of industries whose goods and services are sold within the region. Their market is the region itself. This group will typically include retail, real estate, and similar industries. The region's growth in per capita income and GDP is dependent on the income earned from, and brought into the region by, the basic sector through selling its goods and services outside the region. Thus, the health of the non-basic sector is dependent on the performance of the basic sector.

For the last half of the 20th Century, economic base theory was, and it continues to be into the 21st Century, the most widely used model for analyzing economic growth at the local and regional level. However, also in the last half of the 20th Century, both the Harrod-Domar and Neoclassical models were adapted to explain regional growth. In 1964, Borts and Stien first adapted the Neoclassical model to explain regional growth. And, in 1978, Ghali, Akiyama, and Fujiwara used the Neoclassical framework to study economic growth in the U.S. states. Subsequently, it has been applied to study economic growth at the regional level, not only in the U.S., but also for analyzing regional economic growth in other areas of the world, particularly for the regional economies of the European Union.[9] In 1969, Richardson adapted the Harrod-Domar model to explain economic dynamics at the regional level.

In tracing the development of economic growth theory, from its inception to its application to regional growth and development, two major approaches to economic growth are presented: *Neoclassical growth theory* and its extensions, and *Keynesian-based growth theory*. Very simply, Neoclassical growth theory can be thought of as focusing on problems of supply, while assuming sufficient demand. Keynesian-based growth theories can be thought of as focusing on deficiencies in demand that constrain production (output) and growth. The Harrod-Domar model, a Keynesian-oriented model, focuses on the interaction

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[6] Homer Hoyt was a planner at the U.S. Federal Housing Administration.
[7] Mazilia and Feser (1999, 2000) f. 1, p. 76 and Capello (2007) p. 110. Although, according to the entry in Wikipedia, it was developed by Robert Murray Haig in his work on the Regional Plan of New York in 1928.
[8] ibid, p. 78.
[9] For discussions of applications of Neoclassical Growth Theory to the urban and regional level, see McDonald (1997) p. 303; McCann (2001) Ch. 6; Capello (2007) Ch. 6, and Malizia and Feser (1999, 2000) Ch 3.

between supply *and* demand, and how the interplay and feedbacks between the two drive growth and fluctuations in the economy.[10] Another set of Keynesian-based growth and development models, circular-cumulative causation, developed by Myrdal and formalized by Kaldor, also focuses on the interaction between supply and demand, and how this interaction ignites a chain-reaction process that generates a virtuous circle of cumulative causation propelling a region toward a trajectory of growth and development.

The first step to developing a framework is to retrace developments in growth theory in the Post-World War II (WW II) era, with an emphasis on the roles that knowledge and technology have played in the formulation in each of the growth paradigms. Following Audretsch, Keibach, and Lehmann (2006), the development of economic growth theory in the Post-WW II era is partitioned into three historical periods:[11] Technology as Exogenous: the Solow Model, covered in Section IV; Technology as Endogenous: the Romer-Lucas Model, covered in Section V; and the Entrepreneurial Economy, covered in Section VII. Keynesian/Post-Keynesian perspectives on economic growth are introduced in Section VI. Central to the approach presented here is the treatment of technology and knowledge by each of the growth paradigms, and their implications about the changing role of the entrepreneur and the exploitation of knowledge for the development and introduction of new products and services into the market. Before proceeding, it seems appropriate to provide precise definitions of the terms "technology" and "innovation." ***Technology*** is defined as:

> The sets of *production, organization, information,* and *communications blueprints*, which are available to all firms, and which mediate the relationship between the input factors employed and the output produced.[12]

Innovation is defined as:

> …the adoption and implementation of new production techniques and technologies.[13]

The initial motivation for the development of growth theory after WW II was the Neoclassical response to the Harrod-Domar growth model developed in the 1930's. It is therefore important to frame the context for the advent of Solow and Swan's Neoclassical models of growth. To that end, the Harrod-Domar model is presented in Section III. In addition the adaptation of, and the implications of, the Harrod-Domar model to explain economic dynamics at the regional level are also discussed. For those not familiar with the economic concept of the production function, and for those who would like a brief review, the next section (Section II) reproduces the introduction and review of the production function presented in Appendix A of Volume I[14]. It provides the basis for understanding the subsequent sections of this volume as the evolution of Post-WW II economic growth theory is reviewed. Section IV introduces the Solow-Swan, Neoclassical growth model, and Section V traces the development of endogenous growth theory. This development was motivated by dissatisfaction with the Neoclassical model's treatment of technological progress. In these models, technological progress is endogenous to the growth and development process. Section VI compares and contrasts Neoclassical/endogenous growth approaches to the Keynesian/Post-Keynesian approaches. Surprisingly, though different in their approaches, they yield similar conclusions. Section VII introduces Audretsch, Keilbach, and Lehmann's (2006) endogenous entrepreneurship and regional growth model. Finally, this volume finishes with some conclusions and closing remarks in Section VIII.

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[10] In fact, the Harrod-Domar Model was originally developed to explain the Business Cycle.
[11] This approach follows Audretsch, David B., Max C. Keilbach, and Erik E. Lehmann, ENTREPRENEURSHIP AND ECONOMIC GROWTH (2006) Oxford University Press: New York, Ch. 2.
[12] McCann, Philip, URBAN AND REGIONAL ECONOMICS (2001) Oxford University Press: New York, p. 222.
[13] McCann, p. 222.
[14] Many of the models presented in this volume are predicated on the concept of the *Production Function*, which is introduced in Section II.

II. INTRODUCTION TO THE PRODUCTION FUNCTION

This section reproduces the introduction to the production function that appears in Appendix A, Volume I of this report. What follows provides a brief introduction and review of the production function and some fundamental, derived concepts and relationships. From Robert Solow's 1956 paper on, Neoclassical growth theory has been based on the production function. Thus, a basic grasp of this production relationship is essential for understanding the subsequent discussion of the evolution of economic growth theory since World War II presented in the following sections of this volume.

A. A Functional Relationship

As mentioned above, Neoclassical growth theory is based on the *production function*, a specific instance of the mathematical relationship called a "function." A *function* is a special kind of relation of ordered pairs of numbers, or groups of numbers, such that there is only one value for a corresponding value, or group of values. In the simple case of a functional relationship between two values, there is an *independent* or *input* value, or *variable,* usually denoted as "x," and a *dependent* or *output variable,* denoted as "y." In a functional relationship, any x value *uniquely* determines a value of y. It is also sometimes said that the set of x-values are mapped into the set of y-values. Thus, a function is sometimes called a *mapping* or *transformation*. Symbolically, y is a function of x is expressed as:

$y = f(x)$, which is read: "y equals f of x" (i.e., y is a *function* of x). (II-1.)

The set of all values x can take is called the *domain* of the function, and the set of all values y can take is known as the *range* of the function.[15]

Of particular interest for explaining the production function is the extension of the idea of the function to include two or more variables. The extension to two independent variables can be expressed as:

$$z = f(x,y) \qquad \text{(II-2.)}$$

Now, to determine the value of z, the values of both x and y must be specified. There will be only one value z for every pair of values for x and y.[16] This function is particularly relevant for understanding the *production function*.

B. The Production Function

The production function relates the quantities of capital (K) and labor (L) inputs used to produce a given level of output (Q). Within this context, equation (II-2.) would be re-stated as:

$$Q = Q(K,L) \qquad \text{(II-3.)}$$ [17]

Further, Equation (II-3.) implies that the minimum combination of inputs of K and L were used to produce some given level of output, Q.

For instance, for an auto plant to produce so many lot-runs of 1,000 cars each (Q), it would require a given amount of plant and equipment (K) and workers (L). Thus, within the context of this example, Equation (II-3.) could be re-stated as:

$$\text{1,000 Cars} = Q(\text{Plant and Equipment, Workers}) \qquad \text{(II-4.)}$$

[15] Chiang, Alpha, FUNDAMENTAL METHODS OF MATHEMATICAL ECONOMICS, 3rd Edition (1984) McGraw-Hill: NY, pp. 20-23.
[16] ibid, p. 29.
[17] For some references on an introduction to the production function see Reynolds, R. Larry, *Production and Cost*, BASIC MICROECONOMICS (2000), Call, Steven T. and William L. Holahan, MICROECONOMICS, 2nd Ed. (1983) Wadsworth Publishing: Belmont, CA. Ch. 5, and Mansfield, Edwin, MICROECONOMICS: Theory and Applications, 2nd Ed. (1975) W.W. Norton: New York, Ch. 5.

Thus, the *factor inputs* are plant and equipment (K) and workers (L), and the *output* (Q) is the lot-run of 1,000 cars, which expresses the functional relationship between the two independent, or input, variables, K and L, and the dependent, or output, variable, Q.

Time is an important determinant of the form of a given production function. In the *immediate run*, nothing can be changed. All factors are fixed. In the *short run*, some factor inputs, like labor, can be varied. In the *intermediate-to-long run* all factor inputs are variable. How each of the three perspectives might be defined in terms of the length of time for each depends on many factors, including the industry and the capital intensity of its production process. For the auto plant example above, a period of probably a week would be an immediate-run perspective. A couple of weeks to even months would be a short-run perspective. Clearly, hours could be increased or reduced, and shifts expanded or contracted, as the number of lot-runs is increased or decreased to meet changing market conditions. But, save idling or closing the plant, changing the plant will be a longer time frame perspective. That is, significantly expanding or building a new plant could take up to a couple of years. Thus, the time frame defining each one of the three perspectives would be different for other industries. In the *long run*, not only are all factor inputs variable, but so is technology.

In the immediate-to-short-run, the production function would take the following form:

$$Q_1 = Q(K_o,L_o) \qquad \text{(II-5.)}$$

The above expression conveys the idea that both inputs are fixed in the immediate run for a single lot-run (Q_1) of 1,000 cars.

In the short run, the plant, and probably much of the equipment (K), too, will be fixed, with other factor inputs, particularly labor (L), variable. This is expressed as follows:

$$Q_n = Q(K_o; L) \qquad \text{(II-6.)}$$

Now, Q_n conveys the idea that more than one lot run (i.e., n lot-runs) is being produced, while K_o implies that capital is fixed (i.e., the plant cannot be varied), and that the labor input can be varied (L), that is, it is not fixed in the short run.

In the intermediate-to-long run, *all* inputs are variable. The intermediate-to-long-run, is distinguished from the long run in the way *technology* is specified in the Neoclassical production function. Technology is now introduced into the production function in Equation (II-7.) An expression for an intermediate-run production recipe would include a term for technology being held constant, or fixed:

$$Q = Q(T_o; K,L)^{18} \qquad \text{(II-7.)}$$

Equation (II-7.) indicates that though capital and labor are both variable, technology is held constant, or fixed (T_o). In the long run even technology varies. This is expressed in Equation (II-8.):

$$Q = Q(T,K,L) \qquad \text{(II-8.)}$$

Returning to the auto plant example above, Equation (II-8.) would describe long-run conditions, as the old "Fordest" assembly-line methods were replaced with the introduction of robotics, computerized numerical controlled (CNC) machinery, and the team approach into the auto production process. In this case,

[18] For now, technology (T) is entered into the production function as a third argument. In Section IV, some alternative approaches to representing technology in the Neoclassical production model will be introduced.

technology varied, as the industry adopted information technology-based production techniques, in combination with organizational and process innovations. The new technology could be introduced by building new, "state-of-the-art" plants, and closing older, obsolete facilities, or, if possible, retrofitting existing plants, or some combination of both.

C. Some Features of the Production Function

Several features of the production process arise from the specification of the Neoclassical production function. The first set of points arises from the short-run perspective of production. Recall from above that in the short run, plant size and much of the equipment (i.e., capital, K) and technology (T) are held constant. It is assumed that the variable input over the short run is the labor input. Thus, in the short run, the production function may be expressed in the following form:

$$Q = Q(L) \qquad \text{(II-9.)}$$

Total product (TP) is the total output (Q). That is: TP = Q = Q(L).

Average product (AP) is the output per unit of input, or AP= TP/L = Q/L. In this case, since all other factors and technology are held fixed, and labor is the only variable input, AP = AP_L, which is the *average product of labor*.

Marginal product (MP) is the change in output due to a change in the factor inputs. In this case, since there is only one variable factor input, the marginal product is defined as the *marginal product of labor* (MP_L), which is $MP_L = \Delta TP/\Delta L$ = change in TP/change in the Labor Input (where Δ= change).

Technical efficiency is defined as the ratio of output to input, or output/input. This is distinguished from the AP in that AP is the ratio of output to a *variable input* and a set of *fixed inputs*. For instance, the average product of labor is $AP_L = TP\ (=Q) / \text{inputs}\ (T_o + K_o + L)$, where technology and capital are fixed at T_o and K_o and labor, L, is the variable input. The maximum value of the AP_L is the point where $MP_L = AP_L$, and it represents the *technically efficient* use of the labor input.

D. The Law of Diminishing Returns

The following result is one of the most well known from microeconomics. It will have important implications for the focus of this paper on the evolution of, and the competing theories of, economic growth and development. It applies to the short-run perspective of production.

As a variable input, such as labor, is added to a set of fixed inputs, such as a given size plant, and holding technology constant, output increases rapidly. That is, TP, or Q, increases at an increasing rate. Returning to the auto plant example, eventually, as more and more workers are added to the line of a given auto plant, with a given set of technologies, for a given, short-run, time period, to produce lot-runs of cars, the number of cars produced would begin to level off and grow at a constant rate, and then grow at a decreasing rate. This is because, at some point, additional workers would become redundant. Then, TP, or Q, would be increasing at a decreasing rate. There would be too many workers on the line trying to produce cars in a fixed-sized plant. In general, when more and more of a variable input is added to a fixed input (with technology held constant), for a given time period, output, at first, increases at an increasing rate, then it increases at a constant rate, and eventually at a decreasing rate. If the process is carried far enough, and the variable input reaches the saturation point, relative to the fixed input, then there would actually be negative returns to scale. That is, output (i.e., TP, or Q) would actually start declining. This is known as the ***Law of Diminishing Returns.*** It can be traced back to Ricardo and Mathus.[19] Mansfield notes several points that summarize the assumptions behind the law of diminishing returns:[20]

1. The law of diminishing returns is an empirical generalization, not a deduction from physical or biological laws. In fact, it seems to hold for most production functions in the real world.

2. It is assumed that technology remains fixed. The law of diminishing returns cannot predict the effect of an additional unit of input when technology is allowed to change.

3. It is assumed that there is at least one input whose quantity is being held constant. The law of diminishing returns does not apply to cases where there is a proportional increase in all inputs.

4. It must be possible, of course, to vary the proportions in which the various inputs are used.

III. THE HARROD-DOMAR GROWTH MODEL[21]

In *The General Theory,*[22] Keynes did not extend his theory of demand-determined equilibrium into a theory of growth. This was left for the Cambridge Keynesians to explore. The first to come up with an extension was Sir Roy F. Harrod who (concurrently with Evsey Domar)[23] introduced the "Harrod-Domar" model of growth (Harrod in 1939, Domar in 1946).[24] Although the Harrod-Domar model was initially created to help analyze the business cycle, it was later adapted to explain economic growth.[25] In fact, Domar's model was not intended as a growth model, made no sense as a growth model, and was repudiated as a growth model forty years ago by its creator. So it was ironic that Domar's model became, and continues to be today, the most widely applied growth model in economic history.[26]

The Harrod-Domar model is used in development economics to explain an economy's growth rate in terms of the level of saving and productivity of capital. It suggests there is no natural reason for an economy to have balanced growth. With this brief introduction, the remainder of this section will develop the framework and major implications of the Harrod-Domar growth model.

Within the Keynesian framework, investment is one of the determinants of aggregate demand (AD) and AD is linked to output (or aggregate supply) via the multiplier. Abstracting from all other components, goods market equilibrium is stated as:

$$Y = (1/s)I \qquad \text{(III-1.)}$$

Where: Y = Income = GDP,
I = Investment,
S = Savings = sY
And s = Marginal propensity to save (MPS), and thus the multiplier is $1/s$.

[19] Cannan, Edwin, *The Origin of the Law of Diminishing Returns*, 1813-15, ECONOMIC JOURNAL (1892): 2
[20] Mansfield (1975), p. 128.
[21] For a mathematical presentation, using calculus, see Chiang (1984) pp. 465-469.
[22] Keynes, J.M., THE GENERAL THEORY OF EMPLOYMENT MONEY AND INTEREST (1936)
[23] Harrod, R. F. (1939), An Essay in Dynamic Theory, ECONOMIC JOURNAL, Vol. 49, No. 1 and Domar, D. (1946), Capital Expansion, Rate of Growth and Employment, ECONOMETRICA, Vol. 14.
[24] *Keynesian Growth: the Cambridge version* THE HISTORY OF ECONOMIC THOUGHT WEBSITE http://cepa.newschool.edu/het/ essays/ growth/keynesgrowth.htm accessed on November 2, 2007
[25] *Harrod-Domar Model* WIKIPEDIA, **http://en.wikipedia.org/wiki/Harrod-Domar_model** accessed on June 2007
[26] Easterly, William, *The Ghost of Financing Gap* (July 1997) DRAFT.

The problem is to determine an equilibrium growth rate (g) for the economy. The level of savings is a function of the level of GDP, (i.e., S = sY). That is, savings is equal to the MPS times the level of income. The level of capital (K) needed to produce an output Y is given by the equation:

$$K = \sigma Y \qquad \text{(III-2.)}$$

Where: $\sigma = K/Y$ = Capital-to-output ratio.

Investment is a very important variable for the economy because investment has a dual role. Investment (I) represents an important component of the demand for the output of an economy as well as the increase in capital stock (i.e., supply). Thus $K = \sigma Y$ (Equation III-2).

For equilibrium there must be a balance between supply and demand for a nation's output. In the simple case, this equilibrium condition reduces to I = S. Thus,

$$I = \Delta K = \sigma \Delta Y, \text{ and} \qquad \text{(III-3.)}$$

$$I = S \qquad \text{(III-4.)}$$

So:

$$\sigma \Delta Y = sY. \qquad \text{(III-5.)}$$

Therefore, the equilibrium rate of growth, g, is given by

$$g = \Delta Y/Y = s/\sigma \qquad \text{(III-6.)}$$

Equation (III-3.) states that investment (I) is equal to the change in the capital stock (ΔK), which, in turn, is equal to the capital-to-output ratio (σ) times the change in income, which equals the change in GDP (ΔY). Since, as equation (III-4.) states, investment (I) is equal to savings (S), it follows that the capital-to-output ratio times the change in GDP equals the economy's savings (Sy).

In equation (III-6.), the equilibrium growth rate of output ($g = \Delta Y/Y$) is equal to the ratio of the MPS to the capital-to-output ratio (s/σ). This is a very significant result. It indicates *how the economy can grow such that the growth in the capacity of the economy to produce is matched by the demand for the economy's output.* This is the condition for full employment, *steady state* growth.

Thus, $g = s/\sigma$ is the *warranted rate of growth.* However, Harrod and Domar originally held s and σ as constants that were determined by institutional structures. This gives rise to the famous *razor's edge.* The implication is that if actual growth is slower than the warranted rate, then excess capacity is being generated because the growth of the economy's productive capacity is outstripping the growth in aggregate demand (AD). This excess capacity will induce firms to invest less. The resulting decline in investment will itself reduce demand growth further, causing even greater excess capacity in the next period.

Similarly, if actual growth is faster than the warranted growth rate, then the growth in demand is outstripping the economy's productive capacity. Insufficient capacity implies that entrepreneurs will try to increase capacity through investment, which in turn, further increases demand, making the shortage even more acute. With demand always one step ahead of supply, the Harrod-Domar model guarantees that unless the increase in demand and output are growing at exactly the same rate (i.e., demand is growing at the warranted rate), then the economy will either grow or collapse indefinitely.

The *razor's edge*, thus, means that the steady-state growth path is unstable; the only stable growth path, the *razor's edge*, is where the real growth rate is equal to s/σ permanently. Any slight shock that will lead real growth to deviate from this path ensures that the economy will not gravitate back towards that path, but will rather move further away from it.

The Harrod-Domar model, and its implications, can be summarized as follows. There are three critical concepts related to growth:[27]

1. *Warranted growth* – the rate of output growth at which firms believe they have the correct amount of capital and therefore do not increase or decrease investment, given expectations of future demand.

2. *Natural rate of growth* – The rate at which the labor force expands, a larger labor force generally means a larger aggregate output.

3. *Actual growth* – The actual aggregate output change.

There are two possible problems that are observed in the economy:

First, the relationship between the actual and natural (population) growth rates can cause disparities between the two, as factors that determine actual growth are separate from those that determine natural growth. Factors such as birth control, culture, and general tastes determine the natural growth rate. However, other effects, such as the marginal propensities to save and consume, influence actual output. There is no guarantee that an economy will achieve sufficient output growth to sustain full employment in a context of population growth.

The *second* problem identified in the model is the relationship between actual and warranted growth. If it is expected that output will grow, investment will increase to meet the extra demand. The problem arises when actual growth either exceeds or falls short of warranted growth expectations. A vicious cycle can be created where the difference is exaggerated by attempts to meet the actual demand, causing economic instability. Finally, the major conclusions are:

- Economic growth depends on policies to increase saving (investment), and using that investment more efficiently through technological advances.

- An economy does not find full employment and stable growth rates naturally, similar to Keynesian theory.

Thus, equilibrium in the Harrod-Domar model is a *razor's edge* equilibrium. If the economy deviates in any direction, the result is instability.

Adaptation to Explain Regional Dynamics[28]
In 1969, Richardson adapted the Harrod-Domar growth model to interpret the dynamics of the regional economy. The national version of the model describes the dynamics of a closed economy. But, when

[27] This summary is based on the entry appearing in the on-line encyclopedia, Wikipedia, Harrod, R. F. (1939), An Essay in Dynamic Theory, ECONOMIC JOURNAL, Vol. 49, No. 1 and Domar, D. (1946), Capital Expansion, Rate of Growth and Employment, ECONOMETRICA, Vol. 14, and Chaing (1985) pp. 465-469.
[28] This section follows Capello, Roberta, REGIONAL ECONOMICS (2007) pp. 121-126.

applied to sub-national regions, or regions within a customs union, the Harrod-Domar model must be modified to reflect the dynamics of an open economy, since trade is a substantial part of regional economic activity. Thus, for the model's regional version, the macroeconomic equilibrium condition for the regional economy is:

$$S + M = I + X \qquad \text{(III-7)}$$

Where: X, M = Exports and Imports of capital to, or from, one region to another.

For a given region, i, Equation (III-7) can be re-written as:

$$(s_i + m_i)Y = I_i + X_i \qquad \text{(III-8)}$$

or

$$\frac{Ii}{Yi} = (s_i + m_i) - \frac{Xi}{Yi} \qquad \text{(III-9)}$$

Where: s = MPS
m = Marginal Propensity to Import Capital proportional to income.

And,

$$Yi = \frac{s_i}{v_i} - \frac{\frac{Xi}{Yi}}{v_i} = n_i \qquad \text{(III-10.)}$$

Where: v_i = Investment Accelerator[29] ≡ σ = Capital/Output Ratio
n_i = Population Growth Rate.

The Steady-State in a Regional Open Economy

The implication of Equation (III-10) is that, unlike in the national/closed-economy version, in the regional/open-economy version, capital may grow at the same rate as output (thus, guaranteeing the steady-state) even if investment tends to outstrip savings, provided that the gap between savings and investment is covered by a surplus of net imports. Thus, the regional economic system can, not only finance investments with internally generated savings, but also by importing capital goods from other regions.

Net exports may also help maintain the steady-state equilibrium when there is a surplus of internal savings, because they make up the shortfall between low internal consumption and the level of production required for full capacity utilization

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[29] The Investment Accelerator is $I = v(Y^*_{t+1} - Y_t)$ and $0 \le V \le 1$.

Labor-Market Equilibrium in the Harrod-Domar Regional Framework
Within the regional version of the Harrod-Domar model, the steady-state is maintained by importing capital goods to make up a savings shortfall by an open, regional economy. Similarly, full employment in a region, with an internal shortage of labor, may be maintained by an inflow of workers from other regions. Likewise, an outflow of migrants to other regions may off-set unemployment in the region. The labor-market equilibrium condition is:

$$Y_i = n_i - e_i \qquad \text{(III-11.)}$$

Where: e = Net Migratory Balance (in-migration – out-migration) in each time period as a percent of regional population.

Implications of the Regional Version of the Harrod-Domar Model
Three important implications emerge from the regional/open-economy version of the Harrod-Domar model:

1. The regional version of the Harrod-Domar model results in conditions for constant-rate growth (i.e., steady-state) that is far less restrictive, and therefore more easily sustainable over time than those governing the national/closed-economy version.

However, the steady-state can still be interpreted as the exception rather than the rule. There are no conditions in the model that ensure that there are inter-regional flows of capital and labor sufficient to guarantee growth at a constant rate (i.e., there are no inter-regional flows of production factors that will bring the system into equilibrium).

2. Regions characterized by a net surplus of imports, that is, those for which:

$$m_i = \frac{\sum_j X_j}{Y_i} > 0 \qquad \text{(III-12)}$$

Are regions that grow more rapidly than others (holding the MPS and σ constant). In fact, a net surplus of imports results in a higher growth-rate because the surplus represents extra savings injected into the region from outside (see Equation III-10).

3. If there are differences among the growth-rates of regions, the regional version of the Harrod-Domar model shows that these differences not only persist, but increase, with the passage of time. In fact, when the initial growth-rate of Region i is higher than that of Region j, it follows from Equation (III-10) that:

$\frac{\sum_j X_j}{Y_i}$ where, by definition $\sum_j m_j y_j > \sum_j X_j$ diminishes, giving further impetus to Y_i.

Limitations of the Regional/Open-Economy Harrod-Domar Model
Although the Harrod-Domar model furnishes some useful insights into the regional economy, it was originally developed to explain the dynamics of a national economy[30], and was only subsequently adapted to explain regional growth. Nevertheless, the Harrod-Domar model has offered some useful insights into

[30] And, as noted at the beginning of this section, the Harrod-Domar model was originally developed to explain the business cycle, not economic growth.

the process of regional growth. But, it does have its limitations. Capello[31] identifies three major limitations to the regional version of the Harrod-Domar model:

1. The Harrod-Domar model cannot predict whether inter-regional flows of production factors will restore equilibrium

2. It cannot demonstrate clear tendencies toward divergence, or convergence, among regions

3. Although the Harrod-Domar model correctly predicts that backward regions will be net importers of capital, it provides no explanation as to what determines this greater capacity to attract capital.

IV. TECHNOLOGY AS EXOGENOUS: The Solow Economy[32]

The first period of the Post-WW II era can be characterized as the "Solow Economy." Robert Solow's 1956[33] article was largely addressed to the pessimism about full-employment growth built into the Harrod-Domar growth model (see Section III). Solow's work changed the approach that economists took to study growth. From that point on, the production-function model has been the basis for explaining the determinants of economic growth. The production-function approach relates measures representing two fundamental factors of production: capital (K) and labor (L). (See Section II.) In his 1987 Nobel Lecture [reproduced in GROWTH THEORY: An Exposition, (1987)], Solow stated why his "discomfort" arose over the implications of Harrod and Domar's model:

> Discomfort arose because they worked this out on the assumption that all three of the key ingredients—the savings rate, the rate of growth of the labor force, and the capital-output ratio—were given constants, facts of nature. The savings rate was a fact about preferences; the growth rate of labor supply was a demographic-sociological fact; the capital-output ratio was a technological fact.[34]

Even though these factors could change over time, albeit sporadically and independently, the possibility of steady-state growth would be a stroke of luck. Further, even if steady-state growth were attained, it would be unstable.

Solow then noted that one adjustment could change the outcome predicted by the Harrod-Domar model: allowing a reasonable degree of technological flexibility. This accomplished a few things:

1. The mere existence of a feasible path of steady growth turned out to have wider implications. This allowed for the possibility of a wide range of steady states if there is a wide range of aggregative factor intensities. The variation in factor intensity is probably the most important way the economy can adapt to the Harrod-Domar condition.

2. An implication of diminishing returns is that the equilibrium rate of growth is not only, *not* proportional to the savings (investment) rate, but it is *independent* of the savings (investment) rate. More precisely, the permanent rate of growth of output, per unit of labor input, is

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[31] Capello (2007) p. 126.
[32] For a more detailed presentation of the Solow Growth Model, see Solow, Robert M., GROWTH THEORY: An Exposition, (1987) Oxford University Press: New York and Valdes, Banigno, ECONOMIC GROWTH: Theory, Empirics, and Policy, (1999) Edward Elgar: Northampton, MA. For a mathematical presentation using calculus, see Chiang (1984) pp. 496-501. For an introductory presentation of the basic Solow model, see Jones, Charles I., INTRODUCTION TO ECONOMIC GROWTH (2002) 2nd Ed. W.W. Norton: New York, Ch. 2
[33] Solow, Robert M., *A Contribution to the Theory of Economic Growth*, QUARTERLY J. OF ECONOMICS (February 1956) pp. 65-94
[34] Solow, Robert M (1987), p. x.

independent of the savings (investment) rate and depends entirely on the rate of technological progress in the broadest sense.

3. Earlier growth theory was mechanical or physical in the sense in that it was almost entirely a description of flows and stocks of goods, whereas the Neoclassical growth model describes *equilibrium* paths and works out the price and interest rate dynamics that would support an equilibrium path.

Solow felt that Result 3 brought both good news and bad news to economists concerned with growth theory. The good news: economists like to think in terms of connecting things like the price and interest rate dynamics, and this would get economists interested in growth theory. The bad news: the connection is a bit too pretty and too interesting and unleashes a standing temptation to sound like a very clever Dr. Pangloss.

With this introduction, the focus now turns to Solow's growth model[35]. Recall the production function from Equation (II-3.) in Section II:

$$Q = Q(K,L)$$

Solow began his growth theory model with a special kind of production function called a *constant-returns-to-scale* (CRTS) production function: specifically, the *Cobb-Douglass* production function. For a CRTS production function, if factor inputs (K and L) are doubled, then output (Q) doubles, hence, constant returns to scale. The specific form of the Cobb-Douglas production function is stated in equation (IV-1.):

$$Q = A\,K^{a}\,L^{\,b} \qquad \text{(IV-1.)}$$

Where: A = Multifactor productivity (the *Solow Residual*). It represents the contribution to output (Q) that cannot be accounted for by the factor inputs, capital (K) and labor (L).

a, b = the shares of the capital (K) and labor (L) inputs used in producing output (Q). Each share (*a* for capital and *b* for labor) is less than 1 reflecting diminishing returns of each single factor input, capital and labor, and a + b =1 reflecting constant returns to scale.

Solow noted that any increase in Q could come from one of three sources:

1. An increase in L. However, due to diminishing returns to scale, this would imply a reduction in Q / L or output per worker.

2. An increase in K. An increase in the stock of capital would increase both output and Q / L.

3. An increase in A, or multifactor productivity, could also increase Q / L or output per worker.

To concentrate attention on what happens to Q / L or output per worker (and hence, unless the employment ratio changes, output per capita), Solow rewrote the Cobb-Douglas production function in

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[35] It should be noted that some authors refer to the Neoclassical growth model as the *Solow-Swan* model. Trever Swan independently developed a Neoclassical growth model which was published the same year as Solow's. See Swan, Trever W., *Economic Growth and Capital Accumulation*, THE ECONOMIC RECORD (1956) 32 (November): 334-361.

per capita form. Before stating that form, it will be helpful to demonstrate the implication of the CRTS production function, of which the Cobb-Douglas function is a specific instance.

As stated above, CRTS implies that by multiplying each input by some factor "z," output changes by a multiple of that same factor: Zq = f (Zk, Zl).

Now, if z = 1/L, then:

$$Q*(1/L) = Q[K*(1/L), L*(1/L)] \quad \text{(IV-2.)}$$

or

$$Q/L = Q\ (K/L, 1)$$

Define q = Q/L and k = K/L, so that the production function can now be written as:

$$q = f\ (k) \quad \text{(IV-3.)}$$

Where: q = output per worker
k = capital per worker

Applying this specifically to the Cobb-Douglas formulation:

$$Q / L = A\ K^{a}\ L^{b-1} = A\ K^{a} / L^{1-b} \quad \text{(IV-4.)}$$

Since multiplying by L^{b-1} is the same as dividing by L^{1-b} and, since constant returns to scale is assumed, a + b = 1, and therefore a = 1 – b. The result is:

$$Q = A\ K^{a} / L^{a} = A\ (K/L)^{a} \quad \text{(IV-5.)}$$

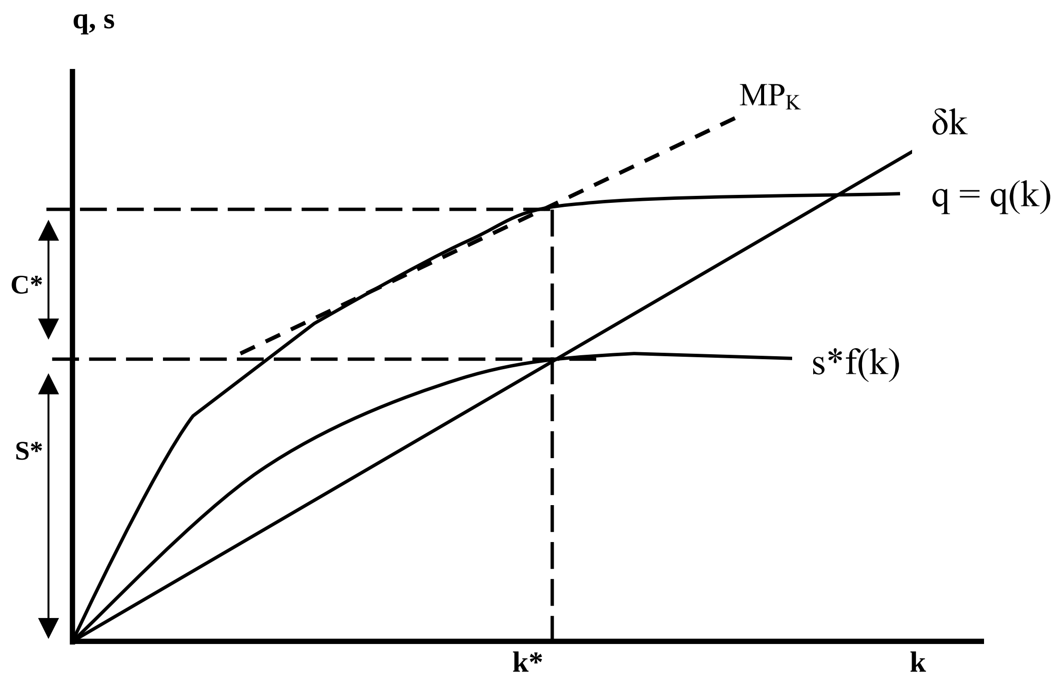
Again, as above, q = Q / L and k = K / L, where lower-case letters equal per capita[36] variables. The result is equation (IV-6.), which is equation (IV-3.) expressed specifically in terms of the Cobb-Douglas production function:

$$q = Ak^{a} \quad \text{(IV-6.)}$$

Graph 1, below, will serve as the vehicle for the following, which introduces some features and implications of the Neoclassical growth model.

[36] It is assumed here that the labor-force participation rate is 1.00 (i.e., 100%). Thus, no distinction would exist between per worker and per capita values.

GRAPH 1: Solow Growth and Capital Accumulation



Growth by Capital Accumulation
The production function shows the production of goods. The focus now turns to the demand for goods, which, in this simple model, consists of consumption plus investment:

$$q = y = c + i \quad \text{(IV-7.)}$$

Where: $y = Q/L = Y/L$
$c = C/L$
$i = I/L$
$Q = Y$ = Aggregate Demand (AD) =Aggregate Supply (AS)

Investment, as always, creates additions to the capital stock. The consumption function in this simple model is:

$$C = (1 - s)\,Y \quad \text{(IV-8.)}$$

Equation (IV-8.) can be rewritten as $c = (1 - s)\,y$, where "s" is the savings rate and $0 < s < 1$. Going back to the demand for goods, $y = c + i$, equation (IV-7.) can be re-written as:

$$y = (1 - s)\,y + i \quad \text{(IV-9.)}$$
$$y = y - sy + i$$
$$\text{and, } y - y + sy = I$$

Thus, $sy = i$: savings equals investment. With these preliminaries, the implications of the growth model can be studied.

To begin with, investment adds to the capital stock (investment is created through savings):

$$i = sy = s\,f(k) \qquad \text{(IV-10.)}$$

The functional relationship expressed in equation (IV-10.) is shown in Graph 1, above. The vertical scale measures per capita output [y = q = q(k)] and per capita savings [s* = f(k)], and the horizontal scale measures the per capita capital stock (k). As reflected by the shape of the function in Graph 1, the higher the level of output, the greater the amount of investment.

Also, it is assumed that a certain amount of capital stock is consumed each period: depreciation takes away from the capital stock. Let "δ" be the depreciation rate. That means that each period δ*k is the amount of capital that is "consumed" (i.e., used up). The depreciation function is the ray coming out of the origin in Graph 1 labeled "δk."

The effects of both investment and depreciation on the capital stock can now be examined.

The growth of the capital stock and the subtraction due to depreciation can be summarized as $\Delta k = i - \delta k$, which is stating that the stock of capital increases due to additions (created by investment) and decreases due to subtractions (caused by depreciation). This can be rewritten as:

$$\Delta k = s^*\, f(k) - \delta k \qquad \text{(IV-11.)}$$

The *Steady State level of the capital stock* is the stock of capital at which investment and depreciation just offset each other: $\Delta k = 0$:

if $k < k^*$ then $i > \delta k$, so k increases towards k*
if $k > k^*$ then $i < \delta k$, so k decreases towards k*

Once the economy gets to k*, the capital stock does not change.

The *Golden Rule level of capital accumulation* is the steady state with the highest level of consumption. The idea behind the Golden Rule is that if policy makers could move the economy to a new steady state, where would they move? The answer is that they would choose the steady state at which consumption is maximized. To alter the steady state, government policy must change the savings rate.

Since $y = c + i$,
then $c = y - i$,
which can be rewritten as:

$$c = f(k) - s\,f(k) \qquad \text{(IV-12.)}$$

which, in the steady state, means $c = f(k) - \delta k$. This indicates that consumption is maximized at the greatest difference between y and depreciation. For those with a background in calculus, to find the point of maximized consumption for $c = f(k) - \delta k$, take the first derivative and set it equal to zero. For those with no calculus background, the important result to remember is that, at the *Golden Rule*, the marginal product of capital must equal the rate of depreciation: $MP_K = \delta$. (See the tangent MP_K in Graph 1, at the point where it is parallel to the δk ray coming out from the origin.)

Population Growth

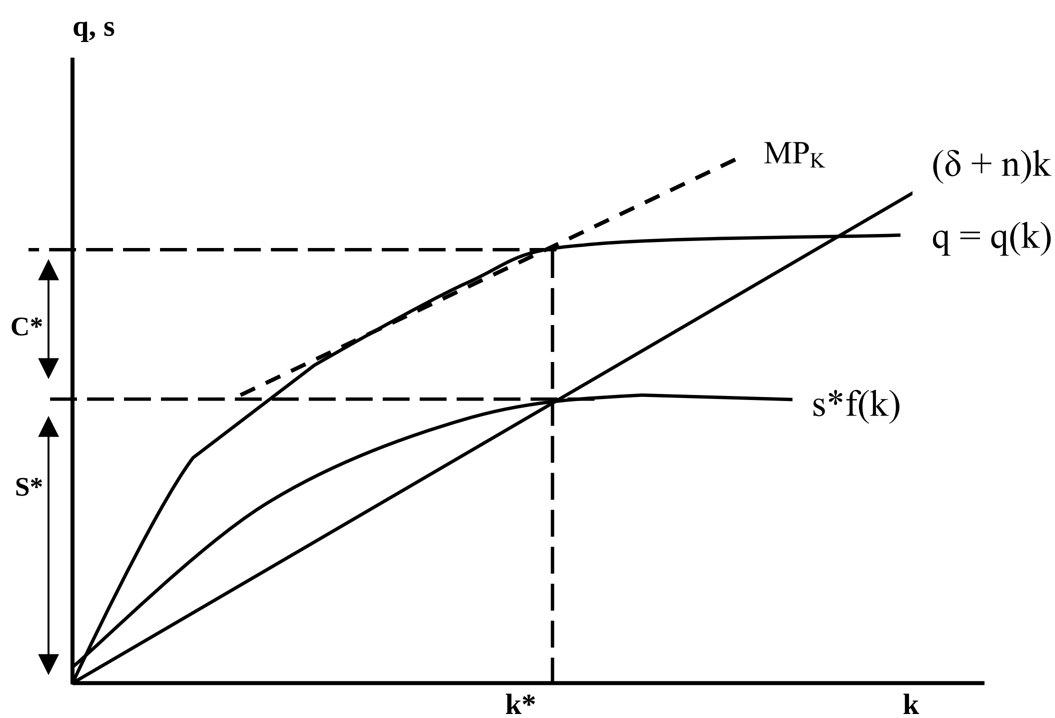
As the labor force (denoted by "n") grows, the capital-to-labor ratio (k = K/L) declines (due to the increase in L), and output per capita (y = Y/L) also declines (also due to the increase in L). Thus, as L grows, the change in k is now:

$$\Delta k = s*f(k) - \delta*k - n*k \qquad \text{(IV-13.)}$$

Where: n*k represents the decrease in the capital stock per unit of labor from having more labor. The steady state condition is now that $s*f(k) = (\delta + n) * k$.

In the steady state, there's no change in k so there's no change in y. This implies that output per worker and capital per worker are both constant. Since, however, the labor force is growing at the rate n (i.e., L increases at the rate "n"), Y (not y) is also increasing at the rate "n." Similarly, K (not k) is increasing at the rate n. Now, at the *Golden Rule*, the marginal product of capital must equal the rate of depreciation, and the growth in the population: $MP_K = \delta + n$. (See the tangent MP_K in Graph 2, at the point where it is parallel to the $(\delta + n)k$ ray coming out from the origin.)

GRAPH 2: Solow Growth and Steady-State, with Population Growth



Technological Progress I: Shifts in the Production Function (Hicks-Neutral Technology)
Solow assumed that technological progress is *exogenous* (i.e., outside the model). Thus, the above analysis assumed that the production function does not change over time. To reflect technological improvement in Solow's model, the production function is modified such that:

$Q = T(t)Q(K,L)$[37] (IV-14)

Where $\Delta T/\Delta t > 0$[38]

Thus, T, some measure of technology, is an increasing function of time. Because of the increasing multiplicative term, T(t), a fixed amount of capital (K) and labor (L) will produce a larger volume of output at a future time period, than in the current time period. This causes an upward shift of the s*f(k) function in Graph 2, resulting in a higher intersection with the $(\delta + n)k$ ray, producing a larger value of k* (the steady-state level of capital per capita). Thus, with technological improvement, there are successively higher steady states with more capital per worker and rises in productivity.

Technological Progress II: Labor Augmenting (Harrod Neutral) Technology
There is an alternative way to introduce technological progress into the Neoclassical model. It can also be assumed that technological progress occurs because of increased efficiency of labor.[39] This assumption can be incorporated into the production function by simply assuming that during each period labor is able to produce more output than the previous period:

$Q = Q(K, L*E)$[40] (IV-15.)

Where: E = Efficiency of labor.

It is assumed that E grows at the rate "g." Still assuming constant returns to scale, the production function can now be written as:

$y = Y / L*E = Q(K/L*E, L/L*E) = q(k)$ (IV-16.)

Where $k = K/L*E$

This now casts the production function in terms of output per efficiency unit of labor and capital per efficiency unit of labor. Since k = K / L *E, k can be examined to see how it changes over time. For those with a calculus background, taking the differential of q(k) yields:

$k\delta - kn - kg$ (IV-17.)

The sign of the first term on the right (kδ) is negative because capital is being consumed by depreciation ($\Delta K/K < 0$). The steady state condition is modified to reflect the technological progress:

$\Delta k = s*f(k) - (\delta + g + n)*k$ (IV-18.)

When $\Delta k = 0$ (i.e., at the steady state), $s*f(k) = (\delta + g + n)*k$. (IV-19.)

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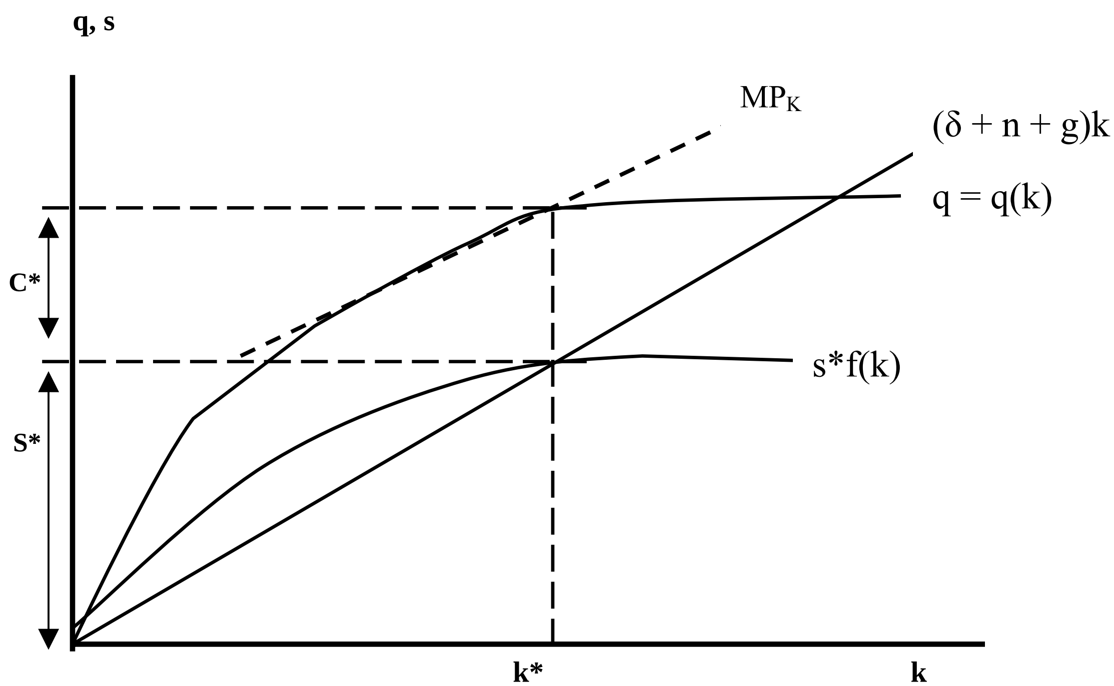
[37] The specification of technology in Equation (IV-14) is known as *Hicks-Neutral* technology. There are several ways of introducing technology into the Neoclassical production function.
[38] Chiang (1984), pp. 499-500.
[39] Uwasu, Michiniori, *The Solow Growth Model* (Spring 2006) APEC 3006
[40] This form of the production function is known as *labor augmenting* or *Harrod neutral* technology. Alternatively, if technology were entered as a multiplicative term with capital (i.e., K*E), then technological change is said to be *capital augmenting* or *Solow neutral* technology [see Valdẽs (1999), pp. 16-19 for a discussion of the different approaches to entering a term for technological change into the Neoclassical growth model].

This condition, which includes population growth and exogenous technological progress, is illustrated in Graph 3. At the steady state, y and k are constant. Since y = Y/L*E, and L grows at the rate n while E grows at the rate g, then Y must grow at the rate n + g. Similarly since k = K/L*E, K must grow at the rate of n + g. The *Golden Rule* level of capital accumulation with this more complicated model is found by maximizing consumption at a steady state, which is expressed in the following relation:

$$MP_K - \delta = n + g \qquad \text{(IV-20.)}$$

This indicates that the marginal product of capital net of depreciation must equal the sum of the rate of population growth population and the rate of technological progress.

GRAPH 3: Solow Growth and Steady-State, with Population Growth and Technological Progress as Increases in Labor Efficiency



Uwasu[41] notes the following points in summarizing the Solow growth model:

1. The Solow model shows how capital stock accumulates over time, determines the long-term equilibrium, and shows how savings, population growth, and technology affect an economy in the long term.

2. Savings and population growth determine the steady state level; however, neither variable explains sustained economic growth.

3. Technological progress can explain economic growth in steady state. In the steady state, output per worker is growing at a rate of technological progress.

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[41] Uwasu (Spring 2006), pp. 15-16.

4. Governments can play a key role in improving the efficiency of workers. Good education/health services, good infrastructure, strict law and order systems, and security improve the efficiency of an economy and thus support sustained economic growth.

5. The rate of technological progress is exogenous in the Solow model. Thus the Solow model itself does not determine sustained economic growth.

Adapting the Neoclassical Model to Explain Regional Growth[42]
The first to formulate a Neoclassical model of regional growth were George Borts and Jerome Stien, in 1964. They applied it to study the growth of U.S. metropolitan areas. Later, in 1978, Ghali, Akiyama, and Fujiwara used the Neoclassical framework to study economic growth in the U.S. states.

The One-Sector Model
The regional version of the Neoclassical growth model makes the usual assumptions:

1. Perfect competition in the goods market

2. Perfect competition in the factor markets, which implies that factor-inputs are compensated in accordance with their marginal productivity, which guarantees profit maximization for the firm

3. Full employment is achieved through flexible factor prices

4. There is costless factor mobility among regions.

5. There is total factor immobility of the goods produced

6. There is perfect substitutability between the two factors [Capital (K) and Labor (L)] in the production of two goods

Since there is no delineation between an export and domestic sector, this version is known as the *One-Sector Model*. As in its national version, regional economic development depends on exogenous technological progress, the growth of the factors of production. The synthesis of these components is embodied in a regional production function, expressed as a regional version of the Cobb-Douglas production function, with constant returns to scale (as introduced above in Equation IV-1). The features of the national/closed version of the Neoclassical growth model, discussed above, apply up to the point at which the consequences of the openness of the regional economy are introduced.

At the regional level, according to Neoclassical growth theory, growth is a matter of the optimal intra- and inter-regional allocation of resources. In an open economy, with perfect factor mobility, a more efficient inter-regional allocation of resources requires the factors of production to migrate to where their productivity is highest, and where they receive their highest compensation.

Therefore, in the regional version of the Neoclassical model, the growth-rate of capital ($\dot{K}$)[43] depends on the amount of internal savings (sY) available to finance investment (I = ΔK), and on the differential between the return on capital in the region (i_r) and the return on capital in the rest of the world (i_w). This is expressed symbolically as:

[42] This section draws heavily on Capello (2007), pp. 135-139.

[43] Where $\dot{K} = \frac{\Delta K}{\Delta t}$ (i.e., the growth-rate of capital over time).

$$\dot{K} = \frac{sY}{K} + \mu(i_r - i_w) \qquad \text{(IV-21)}$$

In the same way, labor grows with the growth of the population (n) and the increase in the differential in wages between the region and the rest of the world (w_r - w_w):

$$I = n + \lambda(w_r - w_w) \qquad \text{(IV-22)}$$

Where: μ, λ = Sensitivity parameters that measure the responsiveness of capital and labor to inter-regional differentials in returns.

Implications for Regional Growth

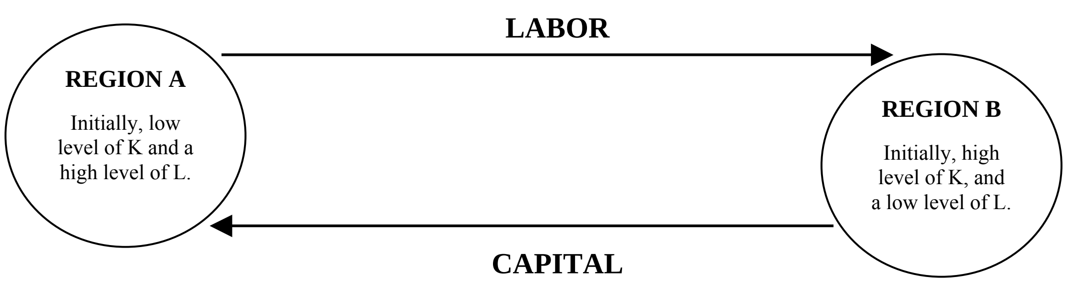
Regional growth, in the Neoclassical, one-sector, growth model is predicated on the outcome of inter-regional flows of the factors of production. To see the outcome, assume two regions: Region A (a poor region) and Region B (a rich region). Initially, Region B has more capital than labor, and Region A has more labor than capital. The one-sector model predicts the following:

1. Capital migrates from the rich region (Region B) to the poor region (Region A)

2. Labor migrates from the poor region (Region A) to the rich region (Region B).

This process is illustrated in Figure 1.

FIGURE 1: Inter-Regional Factor Migration in the One-Sector Neoclassical Growth Model



Driving this inter-regional reallocation of production factors is the differential in factor returns between the two regions. Labor flows from Region A to Region B, where it is more scarce, and therefore commands higher returns. At the same time, the outflow of labor, from Region B, raises the productivity of the labor remaining in Region B, raising their wages. The same mechanism drives the outflow of capital from Region B to Region A. The process stops when the factor returns, factor endowments, and levels of income equalize across the two regions (i.e., the two regions have reached steady-state equilibrium.

When confronted with empirical evidence, the predictions of the one-sector, Neoclassical regional growth model were refuted. After "going back to the drawing board", Neoclassical theorists developed a different approach that more closely conformed to the tendency for capital to flow to regions with higher wages.

The Neoclassical Two-Sector Model of Regional Growth

After Borts and Stien (1968) found that the empirical evidence contradicted the predictions of their one-sector, regional model, when they applied it to U.S. regions, they developed the *two-sector model*. The two-sector model incorporates more realistic assumptions and emphasizes the role the inefficient allocation of resources within a region as the driving force behind intra- and inter-regional factor flows. The result is a dramatically different outcome from that predicted by the one-sector model. In the two-sector model, a reallocation of resources, due to an external shock that moves regions from an initial steady-state equilibrium, pushes local and regional economies toward permanently different growth-rates.

The two-sector model is based on the following assumptions:

1. There are two regions, Region A and Region B:
    a. Each region has two sectors producing two goods, one for export and one for domestic consumption.
    b. The export good (usually manufacturing) is characterized by high productivity, and the domestic good (usually agriculture) is characterized by low productivity.
2. Disequilibria in the trade balance are offset by private capital movements
3. There is perfect competition in the goods market:
    a. Quantities sold by the individual regions do not influence the good's price in the World market
    b. The price of the local (domestic) good is determined by local supply and demand
4. The capital factor-input is used only in the industrial sector (assumption does not affect the final result)
5. There are constant returns in the production of goods
6. Factors are compensated in accordance with their marginal productivities
7. Equality between factor costs and their values of marginal product guarantees profit maximization for firms.

Starting at a point of initial equilibrium, at which there is a stable and uniform growth-rate between regions, and where the growth-rate of capital and labor is constant, and equal to the growth-rate in income, a disturbance is introduced (i.e., an external shock) and the analysis then follows the response to the disturbance of the two-sector model. With this set of assumptions, the model closely resembles the Keynesian/Export-Base model, in which unexplained, exogenous export demand is the source of regional growth.

Suppose that demand for the good exported by one of the two regions (Region A) increases, and as a direct consequence, the price of the good increases, which, in turn, increases the Value of Marginal Product (VMP) of the factors in the region. The two-sector model predicts the following outcome from the intra- and inter-regional reallocation of production resources:

1. The capital stock in the export sector increases due to the inflow of external capital attracted to Region A by higher returns.

2. The increase in the price of the exported good raises the Marginal Product of Labor (MP_L), which increases the demand for labor.

3. The increase in the demand for labor, in conjunction with the consequent increase in wages, attracts workers from the local, regional agricultural sector, and from other regions.

4. The expansion of production and employment in the export sector has a *Backwash Effect* on the agricultural sector, resulting in an increase in the demand for agricultural products, and thus, increases in production and employment.

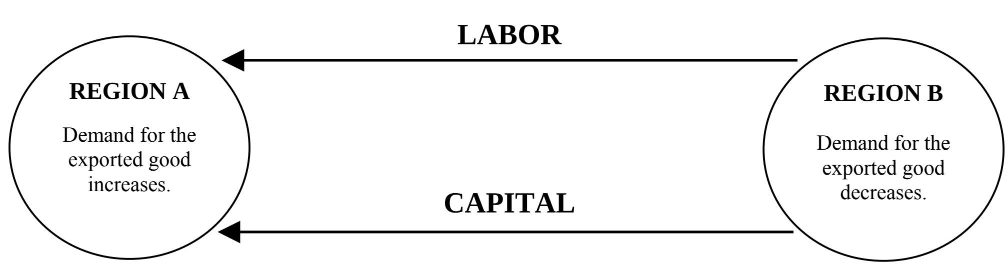
Thus, in the two-sector model, production growth is the result of a more efficient allocation of resources to the manufacturing sector. After the initial stimulus from the increase in demand for the export good, the productive resources in the manufacturing sector are augmented by outside investment, the in-migration of workers from other regions, and the intra-regional reallocation of workers, within Region A, from the agricultural to the manufacturing sector.

Two main conclusions follow from the two-sector model. Interestingly, the conclusions from the Neoclassical two-sector, regional model are in direct conflict with those of the one-sector model. Specifically:

1. Both, capital and labor migrate in the same direction. That is, both capital and labor migrate to the high-wage region (Region A) from the low-wage region (Region B), as depicted in Figure 2. This supports the empirical findings for U.S. regions.

2. The two-sector model demonstrates that there is a tendency for regional growth-rates to diverge. The reason: the income generated in the region exporting the manufacturing good (Region A) differs from disposable income by an amount equal to the return on capital borrowed externally. Internal savings, as a share of disposable income, will therefore, never be enough to finance local production. The shortage of capital guarantees a high return, which stimulates a constant inflow of capital to the region to the region from outside. This results in a persistently higher growth-rate relative to other regions. Further, the divergence in growth-rates is reinforced by the in-migration of workers from other regions, which alters the capital/labor ratio.

FIGURE 2: Inter-Regional Factor Migration in the Two-Sector Neoclassical Growth Model



The agricultural sector acts upon the growth-rate disparities in two ways:

1. It supplies labor to the export sector, and

2. The increase in demand for agricultural goods (i.e., the *backwash effects* discussed above) stimulates production and attracts new workers from outside the region.

The two above results have the effect of mitigating the growth-rate disparities between the export-producing and domestic-producing sectors of the regional economy.

Although the Neoclassical, two-sector model starts with completely different assumptions, and develops within a completely different conceptual framework, it reaches the same conclusion as the regional version of the Harrod-Domar model. That is, both models argue that regions that are net importers of capital will have higher growth-rates. Further, in line with Keynesian models of cumulative causation/development (Verdoorn/Myrdal/Kaldor), the two-sector, Neoclassical growth model demonstrates that this advantage persists over time, exacerbating regional disparities.

Limitations of the Neoclassical Approach

The elegance and rigorous economic logic give the Neoclassical approach great popularity among regional economists, regional scientists, and planners and analysts. Their main merit is attributing a prime role to factor mobility in the regional growth process. This mobility has greater impact at the regional level (as opposed to the national level) because there are fewer spatial and social frictions impeding resource mobility. Nevertheless, there are some significant limitations to the Neoclassical approach. They include some of the following points:

1. Though wealthy regions are highly attractive to labor, the decreasing returns as a consequence of the intense use of labor may diminish their competitiveness. Similarly, backward regions offer locational advantages due to their lower wages and unit-labor costs, and, according to the one-sector model, therefore attract capital. This increases the competitiveness of the backward region.

2. The persistence of marked regional disequilibria suggests that locational advantages are not enough to close the gap between advanced and backward regions. Strong areas are able to absorb decreasing returns that accompany industrialization and high capital-intensity, while weak regions in advanced countries have to compete with the low unit-labor costs characteristic of underdeveloped countries. Backward regions in advanced countries are, therefore, squeezed between the rich, developed countries, and the poor, developing countries.

3. The persistence of regional disequilibria also suggests that migratory flows, as Neoclassical theory interprets them, encounter a number of obstacles in reality. The first, and most obvious, obstacle is the economic and psychological costs of resource mobility. The Neoclassical model assumes away these costs. It, may, in fact, be these costs that explain why the factors do not move in the direction predicted by the one-sector model, or not move at all, for that matter.

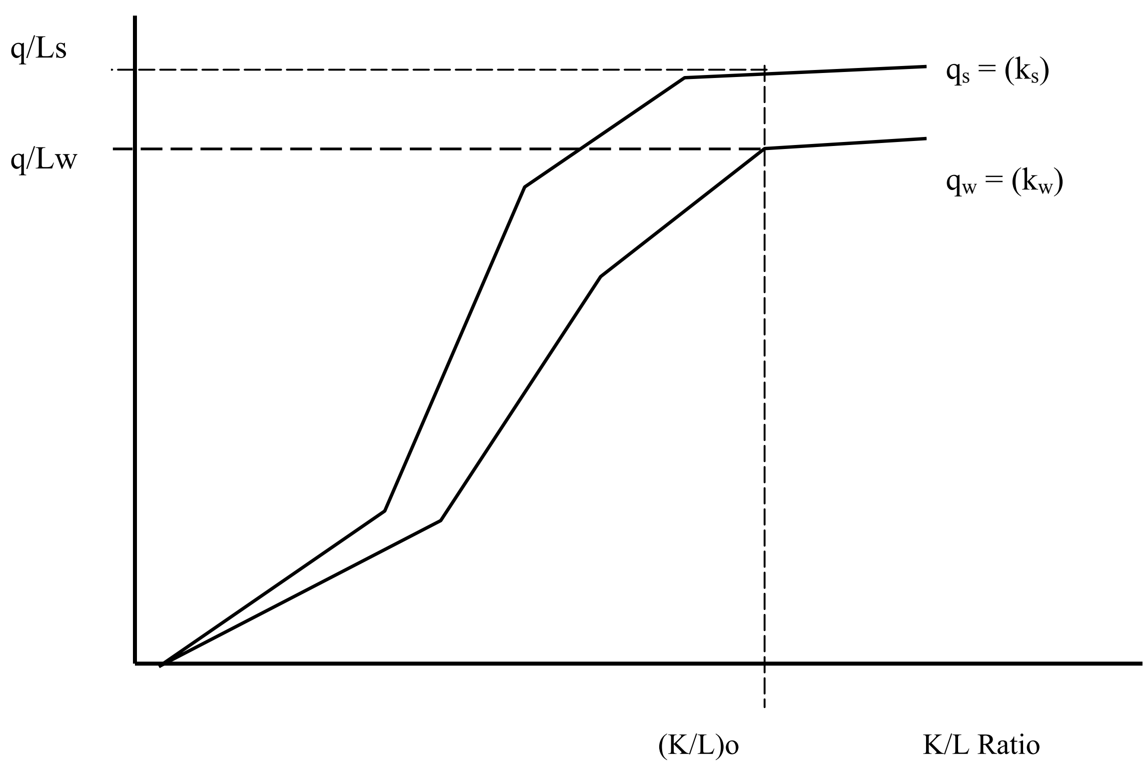
4. Capital tends to remain in rich regions because of cumulative processes and the synergistic process of development. Technical progress in the form of product and process innovations, new knowledge, collective learning, and agglomeration economies in general, induce firms to invest only in rich regions already endowed with capital. These economic advantages are often supplemented by unfavorable social and environmental conditions for productive activities in low per capita-income regions.

5. Labor mobility may also encounter some obstacles. First, the flow of labor to rich regions may well depend on the state of the strong region's economy. That is, migratory flows to the rich region may not take place if it is stagnant, with limited prospects for economic growth. Further, migration from weak to strong regions is often selective. It is often higher skilled workers, who are able to find employment matching their expertise, that are most likely to migrate from a weak to a strong region. Thus, the weak region suffers the out-migration of its more efficient, and skilled resources, which works against convergence. Finally, the existence of labor-market imperfections, which invalidates the perfect-competition assumption, which is at the basis of the Neoclassical logic, may result in wage increases in the presence of unemployment in other regions. In fact, it could exacerbate unemployment.

6. Persistent underdevelopment may be due to the presence of institutional and social impediments to the reallocation of resources to more efficient uses.

7. The uniqueness of the production function for all regions is probably unrealistic. If this assumption is removed, the results of the model change. In the presence of different technologies, and equal capital/labor ratio among regions no longer guarantees an equal level of production. This is illustrated in Graph 4.

GRAPH 4: Production Functions for Two Regions with Different Technologies



V. TECHNOLOGY AS ENDOGENOUS: The Romer-Lucas Economy

In the Harrod-Domar and Solow-Swan growth models (see sections III and IV, above), either explicitly or implicitly, technical change is *exogenous*. That is, technical progress is determined outside the model. This leaves unanswered some critical questions concerning the growth process[44]:

1. The sole long-term determinant of growth, technical progress, is exogenous to the model. Thus, it does not address the capacity of a system to grow.

2. In regard to the diffusion of innovation, the capacity to utilize external and available technical progress differs greatly among regions[45]

3. The model's prediction of regional convergence is at odds with the empirical evidence.

In Schumpeter's growth theory,[46] technical progress is endogenous. Under particular conditions, there would be surges of inventive activity. The Smithian and Ricardian models also had technical change arising from profit squeezes, or, in the particular case of Smith, arising because of previous technical conditions.[47] Nicholas Kaldor was really the first Post-World War II theorist to consider endogenous technical change, in a series of papers, including a famous 1962 paper with J.A. Mirrlees. Kaldor posited the existence of a *technical progress* function, such that per capita income was an increasing function of per capita investment, in which *learning* was regarded as a function of the rate of increase in investment.[48]

Arrow (1962) took on the view that the level of the *learning* coefficient is a function of cumulative investment (i.e. past gross investment). Arrow sought to associate the learning function not with the rate of growth in investment, but rather with the absolute level of knowledge already accumulated. The output of an individual firm is related with capital and labor, as well as the *augmentation* of labor (such as the term "A," which appears in the Cobb-Douglas production function specification of equations IV-4 and IV-6 in Section IV). Arrow (1962) assumed that A, the technical augmentation factor, was not specific to a particular firm, but it is in fact related to total *knowledge* in the economy. This knowledge and experience, Arrow argued, is common to all firms, and a free and public good (i.e. non-competitive consumption). Arrow assumed that the technical augmentation factor is related to economy-wide aggregate capital in a process of *learning by doing*. In other words, the experience of the particular firm is related to the stock of total capital in the economy.

Arrow assumed that increasing only capital (or only labor) does not lead to increasing returns. Rather, capital and labor must both expand. However, by adding this restriction, Arrow's original model exhibits non-increasing returns to scale in aggregate if the rate of growth in an economy is steady. Paul Romer (1986) went to great lengths to disqualify the restriction imposed by Arrow. Taking the Arrow idea of disembodied knowledge, Romer concluded that there indeed could be constant returns. Further, Romer argued, that the rate of growth of capital alone may yield increasing returns.[49]

Endogenous growth models introduce mechanisms internal to the economic processes that explain the growth-rate in per capita output. These mechanisms are identified in non-decreasing returns and in externalities arising from various sources:[50]

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[44] Capello (2007), p. 241.
[45] See SUSTAINED DYNAMISM: Volume I (2008). P. 11 and McCann (2001), p. 224.
[46] Schumpeter, Joseph A., CAPITALISM, SOCIALISM, AND DEMOCRACY (1936)
[47] *Endogenous Growth Theory* Webpage, THE HISTORY OF ECONOMIC THOUGHT WEBSITE, New School University http://homepage.newschool.edu/het/home.htm accessed on November 1, 2007.
[48] ibid
[49] ibid
[50] Capello (2007), pp. 241-242.

1. *Learning-by-Doing*—Accumulated investment in physical capital and the consequent increase in technological capabilities over time.

2. *Increasing Returns to Scale*—There is an aggregate impact of investment by individual firms in the form of positive externalities.

3. *Constant Marginal Return to Capital* (including human capital).

4. *Investment in Human Capital* (scientific and technical knowledge)—Improves the physical productivity of labor.

5. *The Creation of High-Value Intermediate and Final Goods* through investment in R&D to foster innovation, which improves the physical productivity of *all* factors.

The Romer Model of Endogenous Growth: The Knowledge Stock
Romer (1986, 1987) actually proposed two models with two sources of endogenous growth.[51] Romer's first approach assumes that increasing specialization increases output. In turn, output is defined as a function of the number of specialized goods, rather than simply the aggregate capital stock. Given Romer's assumptions, the production function can be expressed as:

$$Q_t = AL^{\beta}K \qquad \text{(V-1.)}$$

Taking the log transformation of equation (V-1.):

$$\ln(Q_t) = \ln(A) + \beta\ln(L) + \ln(K) \qquad \text{(V-2.)}$$

The change over time of the log transformation expressed in equation (V-2.) is:

$$\Delta[\ln(Q_t)]/\Delta t = \beta\{\Delta[\ln(L)]/\Delta t\} + \Delta[\ln(K)]/\Delta t \qquad \text{(V-3.)}$$ [52]

Since the change in a constant over time is zero, $\Delta[\ln(A)]/\Delta t = 0$, where Δt = change, or growth from period t_1 to period t_2, and $\beta = 1 - \alpha$, (the share of the labor input) as in equation (IV-4.) in Section IV.

Equation (V-3.) can be simplified by writing $\dot{Q}$ for $\Delta[\ln(Q_t)]/\Delta t$, the change in Q over time, expressed in equation (V-3.). This allows for a less cluttered symbol than Δt, which as introduced above, and measures change over time (i.e., $\Delta t = t_1 - t_2$). Thus, by letting:

$\dot{Q} = \Delta[\ln(Q_t)]/\Delta t$;

$\dot{L} = \Delta[\ln(L)]/\Delta t$; and

$\dot{K} = \Delta[\ln(K)]/\Delta t$,

Equation (V-3.) can be re-stated in a simplified growth form as:

$$\dot{Q} = \beta\dot{L} + \dot{K} \qquad \text{(V-3a.)}$$

[51] The following draws on McCann (2001), pp. 225-226 and Capello (2007), pp. 242-243.
[52] For those with a calculus background, in instantaneous-rate-of-change form, this is a time derivative expressed as: $d[\ln(Qt)]/dt = \beta\{d[\ln(L)]/dt\} + d[\ln(K)]/dt$.

The argument in equation (V-3.) is that *all* growth is accounted for in terms of the growth of inputs, with the key issue being the level of specialized capital inputs and associated benefits of labor specialization. Critical for regional growth analysis is that one of the underlying arguments for agglomeration economies is that of *increasing location-specific specialization.* If such specialization is, in fact, location- or place-specific, then the endogenous growth model implies that the benefits of this growth will also be localized. Romer also discusses a second potential source of endogenous growth: the stock of knowledge.[53] This is represented by the following production function:

$$Q_t = Q(K,L,E)g(N) \qquad \text{(V-4.)}$$

Where: K = Capital
L = Labor
E = Firm-Specific Knowledge
N = Generally Available Knowledge
Q(•), g(•) = Functional Relationships

If it is assumed that E, N, and K all increase at the same rate, and that Q is a Cobb-Douglas production function, than total output is:

$$Q_t = Q(K,L,E)g(N) = (L^{1-a} K^a)\, K^\psi \qquad \text{(V-5.)}$$

The increasing returns are external to the firm, which maintains a competitive equilibrium. In growth terms, equation (V-5.) becomes:

$$\Delta[\ln(Q_t)]/\Delta t = (1-a)\{\Delta[\ln(L)]/\Delta t\} + \{\Delta[\ln(K)]/\Delta t\}(a + \psi) \qquad \text{(V-6.)}$$

or

$$\dot{Q} = (1-a)\dot{L} + \dot{K}(a + \psi)$$

Equation (V-6.) leads to the following results:

If $(a + \psi) = 1$, then growth is *constant*.

If $(a + \psi) > 1$, then growth is *positive* and *cumulative*.

If $(a + \psi) < 1$, then there is *continuous decline*.

Both of the Romer models conclude that the portion of output growth, considered to be the technology residual (Solow residual) in the Neoclassical growth model, can be attributed entirely to capital acquisition. In Romer's first model, this is because knowledge growth is assumed to increase directly in line with the level of the specialized capital stock. In his second model, Romer assumes that knowledge increases with the level of capital inputs.

Central to Romer's approach is that the externalities generated by technical knowledge, and then embedded in investments accumulated in fixed capital, reach a critical mass, at which point, they take on the characteristics of public goods. At that point, they become available to all firms, whether or not they

[53] This section draws on McCann (2001), p. 226.

participated in the creation of that knowledge. The existence of this "public capital" gives rise to economies of scale in aggregate factor productivity, even though individual factor returns are decreasing.

The Lucas Model of Endogenous Growth: Learning and Human Capital
For Lucas (1988), knowledge inputs also play an important role in economic growth.[54] However, Lucas's focus is on the level of human capital rather then firm-specific capital. He assumes that workers spend a fraction of their time (u) acquiring human capital (H), *learning-by-doing*[55]. Further, it is assumed that human capital increases the productivity of the individual worker. Lucas assumes that there is an *internal* effect, H, and an *external* effect, J, which benefits all other workers.[56] With these assumptions, the production function can be written as:

$$Q_t = (Uhl)^{1-a} K^a J^u \qquad \text{(V-7.)}$$

If it is also assumed that the external, human capital effect (J) is equal to the internal, human capital effect (H), then Equation (V-7.) can be re-written as:

$$Q_t = (Uh^{\theta}L)^{1-a} K^a \qquad \text{(V-8.)}$$

Where: $\theta = (1\text{-}a + \gamma) / (1\text{-}a)$

In order to make growth endogenous, the growth in human capital must be defined as:

$$\Delta H/\Delta t = H^{\rho}v(1-u)^{57} \qquad \text{(V-9.)}$$

or

$$\dot{H} = H^{\rho}v(1-u)$$

Where: ρ, v = Constants, with $\rho \geq 1$ (I.e., there are no diminishing returns to the generation of human capital.)

Taking the simplest case where $\rho = 1$, then the rate of growth of human capital as defined by equation (V-9.) is a constant, λ. Equation (V-9.) can now be re-written as:

$$Q_t = (Ul_q e^{it})^{1-a} K^a \qquad \text{(V-10.)}$$

Where: L_q = Number of units of labor of a given level of efficiency and quality, and is given by $L_q = H^{\theta}\Lambda$.

This implies that a given number of units of labor of increasing human capital can be regarded as equivalent to an increasing number of units of human labor of a fixed efficiency and quality. In growth terms, equation (V-10.) becomes:

$$\Delta[\ln(Q_t)]/\Delta t = (1\text{-}a)(\lambda + \{\Delta[\ln(L)]/\Delta t\}) + a\{\Delta[\ln(K)]/\Delta t\}$$

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[54] This section draws on McCann (2001), pp. 226-227 and Capello (2007), pp. 243-244.
[55] Capello (2007), p. 243.
[56] This would be known a *mixed good* in the Public Finance literature. (See Stiglitz, and Musgrave and Musgrave. Also, see discussion below.) Such a good benefits the individual consuming it, but in consuming the good, the individual generates *positive externalities* that benefit society as a whole
[57] Again, for those familiar with calculus, Equation (V-9.) is the time derivative: $\dot{H}$ = dH/dt = Hρv(1-u).

$= (1\text{-}a)\{\Delta[\ln(L_q)]/\Delta t\} + a\{\Delta[\ln(K)]/\Delta t\}$

$$\dot{Q} = (1\text{-}a)\,\dot{L}_q + a\,\dot{K} \qquad \text{(V-11.)}$$

The implication of equation (V-11.) is that this model concludes that the portion of output growth that is considered to be the technology residual in the Solow Neoclassical model can be attributed entirely to labor through human capital acquisition.

Knowledge: The Critical Factor Input in "New" (Endogenous) Growth Theory

Critical to both the Romer and Lucas models is the internalizing of technological progress within the Neoclassical production-function framework by introducing knowledge as an explicit factor of production. In the Solow growth model, technology was "manna from heaven" that resulted in an upward shift in the aggregate production function (see Section V, above). In contrast, endogenous growth theory sought to identify the mechanism that explained technological progress over time, and to show that it was a product of the internal processes of the economy. New growth theorists challenged the idea that accumulating more and more physical capital was the key to growth, because physical capital is subject to the law of diminishing returns (see Section II, above). New growth theory posits that, unlike physical capital, there are *increasing returns* to knowledge. A key to the increasing-returns characteristic of knowledge is its non-rival, or public good aspect. To see this, Table V-1 presents a classification of goods based on the characteristics that are critical to their being provided by the private market.

TABLE V-1: COMPETITION-CONSUMPTION: A Two-Dimensional Classification of Goods

COMPETITION	CONSUMPTION	
	EXCLUSIVE	NON-EXCLUSIVE
RIVAL	I. Private-Market Goods	II. Mixed Goods
NON-RIVAL	III. Club Goods	IV. Public Goods

SOURCE**:** Based, and expands on, Musgrave and Musgrave (1959)

Table V-1 is based on a two-dimensional classification scheme. Competition for a good could be either rival or non-rival, and consumption of a good could be exclusive or non-exclusive. Goods falling into the Cell I category are goods for which consumers compete. Once the consumer has purchased the good, all others are excluded from consuming the good. An example would be a hamburger. Individual consumers compete for the hamburger (only one person will purchase it). Once the individual consumes the hamburger, all others are excluded from consuming it. Such a good will be produced and allocated by the private market: competition for it is rival, and its consumption is exclusive. In Cell III, Club Goods are non-rival, but consumption is exclusive. These goods are provided by clubs, associations, religious organizations, and similar type groups. Cell IV is the case of Public Goods. Consumption of public goods is non-exclusive and it is non-rival. Once a public good is provided to one, it is provided to all. Adding one more consumer to a public good does not exclude any current or future consumers from consuming it. An example of a public good is national defense.

The private market faces two problems in producing and allocating public goods:[58]

1. Non-excludability gives rise to the *free rider* problem. Because individuals cannot be excluded from consuming a public good, due to non-excludability, there is no effective way of forcing anyone to pay. Some may pay out of a sense of duty, obligation, or altruism, but those who choose not to pay "ride for free."

[58] Based, in part, on Cortright, Joseph, *New Growth Theory, Technology, and Learning* (2001) U.S. Economic Development Administration: Washington, p. 5.

2. Private markets will not produce enough of a public good. Because there is no way a private, for-profit firm can capture revenue equal to benefits received by consumers of public goods, they would not get produced even though it would raise aggregate consumer surplus, and therefore social welfare. This is because firms cannot capture any of the consumer surplus as profits.

For these two reasons, public goods are provided by the government. The most important cell for purposes of the knowledge input and its role in endogenous growth theory is Cell II, Mixed Goods. Mixed goods have the characteristics of both private goods and public goods. That is, competition is rival, but consumption is non-exclusive (see Table V-1). Within the context of endogenous growth theory, the focus here is on education and investment in human capital. Recall Lucas's focus is on the level of human capital rather then firm-specific capital. (See the above discussion on Lucas's model.) He assumed that workers spend a fraction of their time acquiring human capital. Further, he assumed that human capital not only increases the productivity of the individual worker, but that in addition to this *internal* effect, there is also an *external* effect, which benefits all other workers. This is the description of a mixed good (Cell II in Table V-1). An individual certainly must compete for a seat at a postsecondary institution, and training beyond high school certainly raises the lifetime earnings of the individual making the investment in human capital. Thus, there are unmistakable private-market aspects of, and individual private benefits to, education, but there are also positive externalities[59] to this investment in human capital. And the benefit to the whole economy represents the public-good characteristic of education and human capital investment in that others cannot be excluded from the benefits. Hence, it is a mixed good.

Human capital investment is a critical component to providing the intellectual infrastructure for the creation and diffusion of new ideas and knowledge-based products and services (i.e., the creation of intellectual property or wealth), and the ideas themselves have the characteristics of mixed goods. As such, ideas have the following features:

> While they are non-rival—many people can use them at once without depriving others of their use—economically valuable ideas are at least partially excludable. And most importantly, their excludability is more a function of socially determined property rights than it is a function of the intrinsic character of the idea. Patents, trademarks, and copyright law allow individuals to have certain rights to exclude others from the benefits of the ideas they have created. Keeping ideas secret—trade secrets, confidential business information—also allows their owner to exclude others from their benefits.[60]

Thus, ideas are inherently public goods that have been partially privatized by socially determined property rights, rendering them mixed goods. Cortright (2001) gives the following example:

> Because ideas are intangible, when we look at a good like a machine or a service, we don't think about the ideas embedded in it. But digital technologies have sharpened our perception of the difference between ideas and products. Software programs, at their core, long sequences of 1's and 0's encoded in magnetic media, are as close to a pure idea as one can imagine. Software is plainly a non-rival good. The microeconomic analysis of idea production is clear. Because they are non-rival, their marginal cost of production is near zero —the incremental cost of making software available to an additional user is pennies for the diskette and nothing for the program itself.

[59] An *Externality* is generated when a third party, not a party to a transaction, incurs a cost (Negative Externalities) or a benefit (Positive Externalities) as a result of that transaction.
[60] Cortright (2001) p. 5.

> The non-rival quality of ideas is the attribute that drives economic growth. We can all share and reuse ideas at zero, or nearly zero cost. As we accumulate more and more ideas, knowledge about how the world works, and how to extract greater use out of the finite set of resources with which the world is endowed, we enable the economy to develop further.[61]

Thus, it is the public good aspect of knowledge that drives economic growth and development**.**

Limitations of Endogenous Growth Models[62]

A serious drawback to endogenous growth models, especially in explaining the dynamics of regional growth, is their *a-spatiality*. There is no active role for spatial/territorial variables. In fact, whether applied nationally or regionally, there is no modification to the models, as is the case for the Harrod-Domar, Solow-Swan, and Keynesian/Cumulative-Growth models when their original, national-level versions were adapted to explain regional growth.

Attempts to remedy this shortcoming have been implemented through empirical studies on convergence—particularly, β, or conditional convergence[63]. These studies seek to identify socio-economic variables (e.g., human capital, schooling, infrastructure, etc.), which explain why advanced regions obtain higher growth-rates than backward regions—taking into account territorial level. This incorporates the notion that growth is a function of the structural and socio-economic features of the local economy.

VI. KEYNESIAN/POST-KEYNESIAN GROWTH MODELS: Cumulative Causation[64]

The Keynesian/Post-Keynesian approach to regional growth has two major components: Balance of Payments and Regional Growth, and Verdoorn's Law and Myrdal-Kaldor Cumulative Growth.

Inter-Regional Trade and the Balance of Payments

Since this is a Keynesian model, the discussion begins with the standard expression for regional Gross Domestic Product (GDP), which equals aggregate demand (AD), which equals income (Y):

$$GDP = AD = Y = (C + I + G) + (X - M) \qquad \text{(VI-1.)}$$

Where: C = Consumer Spending
I = Business Investment Spending
G = Government Purchases of Goods and Services
X = Exports
M = Imports

The first group of three terms in parentheses, on the right-hand side of equation (VI-1.), is "domestic" regional consumption (spending) or *regional domestic absorption* (A). This represents AD activity within the region. The second group of two terms represents those components of AD associated with inter-regional trade. Equation (VI-1.) can be re-expressed as:

$$(Y - A) = (X - M) \qquad \text{(VI-2.)}$$

[61] Ibid, pp. 5-6.
[62] This section follows Capello (2007), p. 246.
[63] For an introductory discussion of convergence, and the issues surrounding it, see Jones (2002), pp. 63-72. For a more advanced discussion, see Valdès (1999), Chapters 3 and 4.
[64] This section draws from McCann (2001), 228-231 and Capello (2007), pp. 105-121 and 221-227.

Where: $A = C + I + G$
$(Y - A)$ = Net acquisition of assets from other regions; $(X - M)$ = Net exports

Before exploring the implications of differences in regional domestic absorption and the net acquisition of assets from other regions, the discussion begins with a brief presentation of a simple form of the balance-of-payments (BOP) model at the national level. The BOP model can be expressed as:

$$CA_N + FA_N + BOF_N = 0 \qquad \text{(VI-3.)}$$

Where: CA_N = Balance of payments on the national current account
FA_N = Balance of payments on the national financial account (Formally the capital account)[65]
BOF_N = Balance of official payments

The first two terms (from left to right) on the left side of the equation are the net flows of income from economic activity. The CA_N term represents the *current account balance*, which is the balance of trade in goods and services plus the net income from foreign assets owned by domestic citizens. The FA_N term represents the country's net acquisition of foreign assets. Finally, the BOF_N term represents the net difference in the supply and demand for the domestic currency in the foreign exchange markets. Re-arranging equation (VI-3.), it can be expressed as:

$$CA_N + FA_N = -BOF_N \qquad \text{(VI-4.)}$$

If the left-hand side of equation (VI-4.) is positive, then the country is running a BOP surplus, and it is either increasing its stock of foreign assets or reducing its indebtedness to foreign citizens. If it is negative, then the country is running a BOP deficit, and it is either reducing its stock of foreign assets or increasing its indebtedness to foreign citizens. These wealth adjustments are mediated through international currency markets.

In the case of inter-regional trade, all transactions will be mediated in a common currency; thus, there is no official financing, and therefore the BOF_N vanishes at the inter-regional level, as it is equal to zero. This results in modifying equation (VI-4.) to reflect conditions specific to inter-regional trade:

$$CA_R + FA_R = 0 \qquad \text{(VI-5.)}$$

Where: CA_R = Balance of payments on the regional current account.
FA_R = Balance of payments on the regional financial account

Equation (VI-5.) can be rearranged to:

$$CA_R = -FA_R \qquad \text{(VI-5a.)}$$

Thus, the net surplus of a region's trade in goods and services with other regions, given by $(X - M)$ in equation (VI-2.), is balanced by the region's net acquisition of assets from other regions, given by $(Y - A)$ in equation (VI-2.). If a region's export base is strong and growing, then the income generated by the export activity can be used to import more goods and services from other regions, and to buy more assets in other regions. Conversely, if a region is running a BOP deficit, it must be financed by net sales of the

[65]The U.S. Bureau of Economic Analysis made changes to U.S. National Income and Product Accounts such that what was the "Capital Account" is now the "Financial Account."

region's assets to buyers from other regions. If a region's BOP is balanced, then the net acquisition of assets from other regions is zero.

A regional trade deficit cannot go on forever, as a region's assets are finite. Therefore, a region cannot run a long-term BOP deficit. This implies that the level of regional domestic absorption (A), and that of regional income, that can be maintained in the long run, depends on the level of regional exports. In addition, given the level of exports, the long-run growth of regional income is determined by the ratio of the income elasticities of demand of the region's exports to its imports, which depends on the qualitative mix of production sectors in the region. If a region's economic base is concentrated in the production of high value-added, highly income-elastic, and low price-elasticity goods, its export growth will tend to be consistently strong over time for any given pattern of regional imports. If a region's economy is dominated by industries with strong local linkages [i.e., a high *regional purchase coefficient* (RPC)], then its import growth will also tend to be low over time, for any given export pattern. Finally, if a region has an export-base characterized by highly income-elastic exports, in combination with a low, regional income-elasticity of demand for imports, it will tend to have a high, long-run level of growth, even if growth is otherwise muted by a BOP constraint.

Verdoorn's Law and Cumulative Growth[66]

The second component of Keynesian/Post-Keynesian regional growth theory is the role of economies of scale. In 1949, Verdoorn posited that there was a positive relationship between the growth of output and the rate of growth in labor productivity. This is known as *Verdoorn's Law*. The Verdoorn relationship is given by:

$$\dot{p} = a + b\dot{Q} \qquad \text{(VI-6.)}$$

Where: $\dot{p}$ = Growth rate of labor productivity

$\dot{Q}$ = Growth rate of output

Empirical evidence suggests that the value of a is approximately 2%, and that the value of b, the *Verdoorn Coefficient*, is 0.5. These values are broadly consistent with Neoclassical production. By using the notation of the Neoclassical production function (see Section II, above), Equation (VI-6.) can be re-written as:

$$(\dot{Q} - \dot{L}) = a + b\dot{Q} \qquad \text{(VI-7.)}$$

At first glance, Equation (VI-7.) exhibits the econometric, simultaneity problem, because $\dot{Q}$ appears on both sides of the equation. Though there is considerable debate on the issue, Post-Keynesian models assume that the direction of causation is explicitly from right-to-left. That is, increasing output growth generates dynamic economies of scale in production, due to both Arrow's *learning-by-doing* effects on the part of labor, and the increased capital accumulation effects associated with easy credit conditions available during periods of expanding output. Learning-by-doing effects, in combination with increased capital accumulation, result in *dynamic economies of scale*, which, in turn, engenders a regime of *cumulative causation*. The next section now turns to the mechanisms developed by Myrdal and Kaldor to explain the process of cumulative causation.

A Model of Circular and Cumulative Causation: Myrdal and Kaldor[67]

In 1957, Myrdal formulated a model that ran counter to the Neoclassical assumption that processes tend toward equilibrium. His model of circular and cumulative causation was consistent with the empirical

[66] This section follows McCann (2001) pp. 235-239.
[67] This section follows Capello (2007), pp. 221-227.

evidence, which showed persistent inter-regional disequilibria, in terms of self-reinforcing virtuous, or vicious, circles. According to Myrdal's model, if market forces alone are left to operate, rich regions will get richer, and poor regions will get poorer. These results, at odds with the Neoclassical model, follow from the assumptions behind Myrdal's model:

1. An investment function which is based on *Accelerator Theory* (where investment is driven by the real, or expected, level of demand), rather than on the Neoclassical assumption of the rate of return on capital.

2. Economies of agglomeration which are generated by the spatial concentration of productive activities, and by the accumulation of knowledge embodied in capital goods, as opposed to the Neoclassical assumption of constant returns.

This was the first growth model to hypothesize that increasing returns plays a role in the local economic-development trajectory. It also removed the restrictive Neoclassical assumption of a single production function, which implies that there is equal technology across regions. Under these hypotheses, two virtuous processes operate in strong regions:

1. They attract workers because they have a high level of production, and therefore a strong demand for labor. In addition, unlike Neoclassical theory, which assumes that labor is homogenous, the Myrdal model assumes that there is a selective migratory process, in which highly-skilled labor (high levels of human capital) migrates to the strong region, depleting the weak region of their skilled workers. The migratory flows to the strong region expand the local market, stimulate new investments, and attract new capital in a virtuous circle of development.

2. At the same time, the close concentration of production activities in a particular area generates agglomeration economies, which act upon the region's productivity and competitiveness, boosting development. Greater supply generates further labor demand, increased, internal and external, demand for locally-produced goods, new investments, new business start-ups, closer concentration of activities, greater advantages deriving from concentrated locations, and consequent, further productivity increases, in a virtuous demand/supply circle.

Conversely, the reverse processes of emigration, capital loss, decreasing internal demand, and a decline in productivity due to diminished agglomeration economies characterize the poor region. According to the logic of Myrdal's model, the poor region is bound to suffer desertion and poverty.

However, the Myrdal model sets limits on the infinite evolution of the circular-cumulative process; limits based on territorial and supply-side factors. According to Myrdal, a constant and concentrated development process generates *spread effects* (i.e., Myrdal's term for diffusion effects) due to physical congestion, the growing scarcity of production factors, and their increasing costs. These diffusion costs may arise in a region because of spatial contiguity and then spread along transport and communication axes, or they may *filter down* through the branches of the urban hierarchy.

In 1970, Nicholas Kaldor formalized Myrdal's model of circular-cumulative causation[68]. In Kaldor's formal model, dynamic local income ($\dot{Y}_R$) is dependent on the growth of exports ($\dot{X}_R$). Exports exhibit a dynamic, which depends partly on exogenous factors connected with the development of the World economy ($\dot{Y}_W$), and partly on endogenous elements connected with the trend of local competitiveness, which depends on domestic price variation (p). In turn, domestic price variation is explained by the

[68] Kaldor, N., *The Case of Regional Policies*, SCOTISH JOURNAL OF POLITICAL ECONOMY (1970) (3): 337-348.

variation in the cost per labor-unit of output termed the *Efficiency Wage*. Therefore, the efficiency wage is the difference between the growth-rates of wages ($\dot{w}$) and productivity ($\dot{Q}$).

Finally, productivity growth is governed by Verdoorn's Law (discussed above), which states that the growth-rate in productivity consists of an exogenous component (d) and an endogenous component expressed as the growth-rate of output. This last relation states that more than proportional productivity growth-rates are associated with higher growth-rates in output. This relation is explained by scale economies and by learning-by-doing effects (i.e., dynamic economies of scale, again, see above discussion of Verdoorn's Law), and they are captured by the positive parameter (f) of the expression:

$$\dot{Y}_R = a\,\dot{X}_R \qquad \text{(VI-8)}$$

$$\dot{X}_R = \dot{Y}_W - cp = \dot{Y}_W - c(\dot{w} - \dot{Q}) \qquad \text{(VI-9)}$$

Where: $\dot{Y}_W > 0$, $c > 0$, $\dot{w} > 0$.

$$\dot{Q} = d + f\dot{Y}_R \qquad \text{(VI-10)}$$

Where: $d > 0$, $f > 0$.

According to the formal model, the economic conditions that determine a trajectory of growth, rather than decline, are the following:

1. A greater elasticity of demand for the region's exports (parameter a).
2. The higher the increasing returns of output-growth due to productivity growth (parameter f).
3. The greater the elasticity of exports to the variation in productivity, and in domestic prices (parameter c).

Likewise, a weak regional economy, which suffers from weak regional conditions, will exhibit the following:

1. A low initial rate of output growth ($\dot{Y}_R$).
2. Limited growth of the exogenous component of productivity (d) and competitiveness ($\dot{Y}_W$).

The result is that a vicious circle of decline ensues, even if there is parity in the endogenous conditions represented by equal values of the parameters a, c, and f. [69]

Keynesian/Post-Keynesian and Neoclassical/Endogenous Growth Models: Similar Conclusions[70]
The Keynesian/Post-Keynesian growth models, whether the Harrod-Domar type, or the Myrdal-Kaldor model, differ fundamentally, in their basic assumptions, from the Neoclassical/Endogenous-Growth approach to regional growth. In particular, the Keynesian/Post-Keynesian approaches do not require that factor inputs be paid according to their marginal products. Further, they do not require the restrictive

[69] For a graphical presentation of the two sets of results from Kaldor's model, see Capello (2007), pp. 225-227.
[70] This section follows McCann (2001) pp. 238-239 and Capello (2007), p. 141.

assumption that production exhibits constant returns to scale, with respect to factor inputs. However, there are some similarities with models of endogenous growth. Both models imply that there is no particular long-run rate of growth towards which a region is expected to converge. The actual regional growth rates will depend on the extent to which economies, or diseconomies, of agglomeration are operative. And, in terms of regional growth, even Keynesian/Post-Keynesian and Neoclassical models have been shown to produce largely equivalent results. Particularly, empirical observation of regional development can be given interpretations that are consistent with either of the approaches.

There is a principle in the social sciences called *Triangulation*[71]. It refers to the instance in which at least, three different approaches yield the same or similar results. In that case, the three different methods can be viewed as cross-validating the results. Denzin (1978) identified four different types of triangulation. *Theory Triangulation,* which involves using more than one theoretical scheme in the interpretation of the phenomenon, in this case, the regional growth process, would seem to be the most appropriate description of the form suggested here. That is, the similar outcomes predicted by the Harrod-Domar, Keynesian/Post-Keynesian, circular-cumulative, and Neoclassical/Endogenous growth models can act as a cross-validation via theory triangulation, not only because all three predict similar outcomes in the regional and inter-regional growth process, but in addition, empirical studies have supported the notion that regional growth-rates do not converge, an outcome predicted by all three models.

Finally, Harrod-Domar, Keynesian/Post-Keynesian, circular-cumulative, and Neoclassical/Endogenous growth theories all offer different perspectives on the regional growth process. They, therefore, offer different insights into how some regions grow, while others either stagnate, decline, or both. And, the more perspectives that economists, planners, policymakers, and other interested observers have on the regional growth process, the better their understanding of the dynamics driving the regional economy.

VII. A MODEL OF ENDOGENOUS ENTREPRENEURSHIP AND REGIONAL GROWTH

In their Theory of Endogenous Entrepreneurship, Audretsch, Keilbach, and Lehmann (2006)[72] emphasize the critical delineation between *information* and *knowledge*. While advances in information technology have rendered the cost of transmitting information across space trivial, the cost of transferring knowledge across space still follows a steep decay function. The consequence of this steep decay function is the localization of knowledge spillovers. Why would the transmission of knowledge be spatially bounded, when the transmission of information is not? The answer lies in the different characteristics of information, as opposed to knowledge. Information has a singular meaning and interpretation, it can be codified at low cost, and the transactions costs are trivial. On the other hand, knowledge is vague, difficult to codify, and, often, only serendipitously recognized. In addition, localization theories suggest that face-to-face communication and non-verbal cues facilitate the transmission of ideas and intuition that cannot be communicated through codified instructions. Further, information is often context free, while knowledge, especially tacit knowledge, is often derived from specific contexts.[73] Consequently, geographic proximity is critical to the transmission of knowledge—especially, tacit knowledge.
In addition to the distinction between information and knowledge, also critical to the development of a model of entrepreneurship and growth, is the idea that there is a barrier to translating new knowledge into new *economic* knowledge. Audretsch, et al (2006) formalize this idea in their concept of the *knowledge filter.*[74]

[71] See *Triangulation (social science),* WIKIPEDIA http://en.wikipedia.org/wiki/Triangulation_(social_science) accessed on June 30, 2008
[72] Audretsch, David B., Max C. Keilbach, and Erik E. Lehmann, ENTREPRENEURSHIP AND ECONOMIC GROWTH (2006) Oxford University Press: New York, pp. 20-23.
[73] Audretsch, Keilbach, and Lehmann (2006), pp. 22-23.
[74] ibid, p. 41.

Because of high uncertainty, asymmetries, and transactions costs that are inherent in knowledge, decision-making hierarchies may decide not to pursue and commercialize new ideas that an individual, or group of individuals, may think are potentially commercially valuable. These basic conditions of new knowledge, combined with a broad range of institutions, rules, and regulations, result in the *knowledge filter*, or the gap between new knowledge and Arrow's economic or commercialized knowledge. According to Audretsch, Keilbach, and Lehmann (2006), not only is the knowledge filter the consequence of the basic conditions inherent in new knowledge, but it is also what creates the opportunity for entrepreneurship in the *knowledge spillover theory of entrepreneurship.* In this theory of entrepreneurship, the fundamental decision-making unit of observation in the model of the production function is shifted away from exogenously assumed firms to individuals such as engineers, scientists, or other knowledge workers.

Challenging the Assumptions of Endogenous Growth Models

Audretsch, et al's (2006) knowledge spillover theory of entrepreneurship challenges two of the fundamental assumptions implicitly driving the results of the endogenous growth models:[75]

1. The assumption that knowledge is automatically equated with *economic* knowledge. Arrow (1962) has emphasized that knowledge is inherently different from the traditional factors of production, resulting in a gap between knowledge and economic, or commercialized, knowledge.

2. The assumption of knowledge spillover. The existence of the knowledge factor input is equated with its automatic spillover, yielding endogenous growth. In the knowledge spillover theory of entrepreneurship, the *knowledge filter* imposes a gap between new knowledge and new *economic* knowledge and results in a lower level of knowledge spillovers.

Further, they argue, as a result of the knowledge filter, entrepreneurship becomes central to generating economic growth by serving as a conduit for knowledge spillovers.

The Formal Model of Endogenous Entrepreneurship[76]

The starting point for models of economic growth in the Solow tradition is that the rate of technical change (i.e., the rate at which new technical knowledge is created) is exogenous. This view has been challenged by endogenous growth theory. In Romer's growth model, the production function is expressed as:

$$Q^{77} = K^{\alpha}(AL_Q)^{(1-\alpha)} \qquad \text{(VII-1.)}$$

Where: Q = Output
K = Capital stock
A = Stock of knowledge capital

The capital accumulation function is standard from the Solow model:

$$\dot{K} = s_K Y - \delta K \qquad \text{(VII-2.)}$$

Where: s_K = Savings rate and Y = Income.
δ = Depreciation rate of capital[78]
$\dot{K}$ = Change in the capital stock over time (see Section V, above)

[75] ibid, p. 43.
[76] This section follows Audretsch, et al's (2006) presentation of their formal model (pp. 44-46).
[77] Audretsch, et al's (2006) denote output as "Y". Here "Y" is replaced with "Q" for consistency.
[78] Audretsch, et al (2006) use the symbol "Δ" for depreciation of capital, but in this paper, the symbol "δ" is used for capital depreciation, for consistency, it is used to indicate depreciation here, too.

The R&D sector is modeled as:

$$\dot{A} = \rho^* L_A \qquad \text{(VII-3.)}$$

Where: $\dot{A}$ = Change in the stock of knowledge capital over time.
ρ[79] = *Discovery rate* of new inventions, with

$$\rho^* = \rho L_A^{1-\lambda} A^{\varphi} \qquad \text{(VII-4.)}$$

Where: L_A = Amount of labor active in the generation of new knowledge (such as R&D Personnel).
λ = Returns to scale in R&D.
φ = Parameter that expresses the intensity of *knowledge spillovers*

By inserting equation (VII-4.) into equation (VII-3.), the rate of creation of new knowledge is obtained (the *rate of endogenous technical change*):

$$\dot{A} = \rho L_A^{\lambda} A^{\varphi} \qquad \text{(VII-5.)}$$

In the Romer, Lucas, and Jones models, knowledge automatically spills over and is commercialized, reflecting the Arrow observation about the non-excludability and non-exhaustive properties of new knowledge. Thus, investments in R&D and human capital automatically affect output in a multiplicative manner because of their external properties, suggesting the new knowledge, A, is tantamount to commercialized economic knowledge, A_C, that is: $A = A_C$.

Audretsch, et al argue that the non-excludability and non-exhaustible properties are better suited for information than for knowledge. In contrast, they point to Arrow's argument that there is a gap between new knowledge and what actually becomes commercialized (i.e., new *economic* knowledge: $A - A_C > 0$). They define the *knowledge filter* as the gap existing between investments in knowledge and the commercialization of knowledge, or economic knowledge. Letting θ denote the knowledge filter:

$$\theta = A_C / A \quad \text{with: } 0 \le A_C \le A. \quad \text{Hence: } \theta \in [0,1] \qquad \text{(VII-6.)}$$

Where: θ = *Permeability* of the knowledge filter.

It is the existence of the knowledge filter, or knowledge not commercialized by incumbent firms, that generates the entrepreneurial opportunities for commercializing knowledge spillovers. As long as incumbent firms cannot exhaust all of the commercialization opportunities arising from their investments in new knowledge, opportunities will be generated for potential entrepreneurs to commercialize that knowledge by starting a new firm. Audretsch, et al, express the actual level of new technological knowledge used by incumbent firms as:

$$A_C = \theta \bullet \rho L_A^{\lambda} A^{\varphi} \qquad \text{(VII-7.)}$$

Correspondingly, the remaining *untapped* part $(1 - \theta)$ is *opportunities* (Opp) that can be taken on by new firms. They denote this part *entrepreneurial opportunities*, expressed as:

[79] For the same reasons as cited above, Adreusch, et al's use of the symbol "δ" for the *Discovery Rate* is replaced here with the symbol "ρ."

$$\dot{A}_{Opp} = (1 - \theta)\dot{A} = (1 - \theta) \bullet \rho L_A{}^{\lambda} A^{\varphi} \qquad \text{(VII-8.)}$$

The observation that knowledge conditions dictate the relative advantages in taking advantage of opportunities arising from investments in knowledge of incumbents versus small and large enterprises is predicated on the distinction between two knowledge regimes: the *routinized technological regime* versus the *entrepreneurial technological regime*. The routinized technological regime reflects knowledge conditions where the large incumbent firms have the innovative advantage. Conversely, in the entrepreneurial technological regime, knowledge conditions give the advantage to small firms.

Audretsch, et al, then emphasize two important distinctions:

1. In the entrepreneurial regime, the small firms exist and will commercialize the new knowledge or innovate. However, within the context of the spillover theory of entrepreneurship, the new firm is *endogenously created*, via entrepreneurship, motivated by the recognition and pursuit of an opportunity, by an individual, or a group of individuals, in an attempt to appropriate the value of that knowledge.

2. In the knowledge spillover theory of entrepreneurship, the knowledge filter impedes and preempts at least some of the knowledge spillover and commercialization of knowledge. Only select spillover mechanisms, such as *entrepreneurship*, can permeate the knowledge filter. But this is not a forgone conclusion, as the situation will vary across specific contexts and depends on a broad range of factors spanning individual characteristics, institutions, culture, and laws, and is characterized by what Audretsch, et al (2006) call *entrepreneurship capital*.[80]

To merely explain entrepreneurship as the residual from $\dot{A}_{Opp} = \dot{A} - \dot{A}_C$ assumes that all opportunities left uncommercialized will automatically result in the commercialized spillover of knowledge via entrepreneurship. However, the capacity of each regional economy to generate entrepreneurial spillovers and commercialize knowledge is not the same. And, just as the knowledge filter should not be assumed to be impermeable, the capacity of a region's economy to generate knowledge spillovers via entrepreneurship to permeate the knowledge filter should not be assumed to be automatic. It depends upon a region's capacity to generate an entrepreneurial response that permeates the knowledge filter and creates a conduit for transmitting knowledge spillovers.

To explore this process Audretsch, et al (2006) formulate a model of the cognitive process of recognizing and acting on perceived opportunities that flow from knowledge spillovers and other sources. It is formally stated in the following expression:

$$E = f(\pi^* - w) \qquad \text{(VII-9)}$$

Where: E = The decision to become an entrepreneur.
π^* = Expected profit from starting a new firm.
w = Expected wage from employment at an existing firm.

In regard to the exact sources of entrepreneurial opportunities, which are predicated on expected profits from a new-firm, start-up, the focus of theoretical and empirical work has been on individual characteristics, such as attitudes toward risk and access to financial capital, and the existence of social capital. Thus, entrepreneurship is a function of the variation in individual characteristics, holding context constant. Thus, in holding context constant, the implication is that individuals differ across contexts. Audretsch, et al, invert this approach by holding individual characteristics constant, and allowing context

[80] ibid, Ch. 4.

to vary. In other words, if the characteristics of the individual are held constant, how do differences in context affect the individual's behavior? Specifically, if the knowledge context varies across regions, then would an individual with a given set of characteristics, be more inclined to make the entrepreneurial choice in a region characterized by a rich knowledge context, than in a knowledge-poor region? There are, of course, other factors influencing the decision to become an entrepreneur. Access to a context rich in knowledge-spillovers is not mutually exclusive with other factors influencing the entrepreneurial decision. In fact, in line with the circular-cumulative causation models, discussed in Section VI, growth, especially unanticipated growth, is another major contextual variable. Given that, Equation (VII-9) can be re-written as:

$$E = f(\pi^*[g\gamma,\ \dot{A}opp,\ \theta] - w) \qquad \text{(VII-10)}$$

Where: $g\gamma$ = Expected profits arising from general economic growth.

$\dot{A}opp$ = Expected profits arising from potential knowledge spillovers.

θ = The knowledge filter.

Equation (VII-10) states that expected profits are based on opportunities that arise from general economic growth ($g\gamma$), and from potential knowledge spillovers ($\dot{A}$ opp). Thus, expected profits from an entrepreneurial start-up can be decomposed into two major parts, identified in Equations (VII-11) to (VII-13):

$$E = \overline{E} + E^* \qquad \text{(VII-11)}$$

Where: $\overline{E} = f(\pi^*[g\gamma] - w)$ (VII-12)

And,

$$E^* = f(\pi^*[\dot{A}opp] - w) \qquad \text{(VII-13)}$$

The decision to become an entrepreneur (E) is partitioned into two parts: $\overline{E}$, non-knowledge sources, such as general economic growth, and E*, knowledge-spillover sources.

Anticipated growth will most likely be met by incumbent firms as they invest and expand their capacity to meet the increase in demand. However, there may be constraints on existing firms inhibiting them from expanding capacity to meet unexpected increases in demand. It is this unexpected growth in demand ($g\gamma$) that has the potential to generate entrepreneurial opportunities that have nothing to do with new knowledge (Equation VII-12). Thus, there is a distinction between starting a traditional business (e.g., opening up a restaurant, starting a landscaping business) and an entrepreneurial venture in which the new firm start-up introduces a new product into the market, a new process innovation, or products based on a new technology, or some combination of these. These start-ups are more characteristic of entrepreneurship based on knowledge spillovers and technology transfer. There are two sources that shape this type of entrepreneurial activity:

1. The amount of new knowledge being produced, and

2. The permeability of the knowledge filter, which limits the commercialization of new knowledge by existing firms.

If there were neither new knowledge nor ideas being generated, then there would be no spillover opportunities for potential entrepreneurs to capture in a new firm start-up. Entrepreneurship might be triggered by other factors, but not by knowledge opportunities. Similarly, in the absence of the knowledge filter, all opportunities for appropriating the value of knowledge would be pursued and commercialized by incumbent firms.

Thus, two factors shape the relative importance of knowledge-spillover entrepreneurship:

1. The amount of investment in creating new knowledge ($\dot{A}$), and

2. The magnitude of the knowledge filter (θ).

Thus, knowledge-spillover entrepreneurship (Equation VII-13) is the attempt to appropriate profit opportunities accruing from the commercialization of knowledge not commercialized by existing firms (i.e., 1 – θ). However, Equation VII-13 ignores the fact that some regions may have institutional, financial, cultural, and individual barriers to entrepreneurship. These barriers are denoted by Audretsch, et al (2006) as "β". The greater the value of β, the greater the barriers to entrepreneurship. It is the existence of these barriers that explain why some individuals may decide not to become entrepreneurs, even when endowed with knowledge that would otherwise generate a potentially profitable entrepreneurial opportunity. Since the total amount of entrepreneurial activity exceeds that generated by knowledge spillovers (E > E*), Equation VII-10 may be restated as:

$$E = \frac{1}{\beta} f(\pi^*[g\gamma,\ \dot{A}\, opp,\ \theta] - w) \qquad \text{(VII-14)}$$

Equation VII-14 leads to two propositions[81]:

- *Entrepreneurial Opportunities Proposition:* Entrepreneurship will be greater in regions with a greater amount of non-knowledge entrepreneurial opportunities, such as growth.

- *Barriers to Entrepreneurship Proposition:* Entrepreneurship will be lower in regions burdened with barriers to entrepreneurship.

The Determinants of Entrepreneurship and Their Impact on Economic performance: Six Hypotheses
Based on the discussion above of Audretsch, et al's model of the knowledge spillover theory of entrepreneurship and economic growth, and their framework for analyzing the recognition of and then acting upon entrepreneurial opportunities, Audretsch, et al derive the following hypotheses concerning the determinants of entrepreneurship and their Impact on economic performance:[82]

1. *Endogenous Entrepreneurship Hypothesis*: Entrepreneurship will be greater in the presence of higher investments in new knowledge, ceteris paribus.

2. *Economic Performance Hypothesis*: Entrepreneurial activity will increase the level of economic output since entrepreneurship serves as a mechanism facilitating the spillover and commercialization of knowledge.

3. *Location Hypothesis:* Knowledge spillover entrepreneurship will tend to be spatially located within close geographic proximity to the source of knowledge actually producing that knowledge.

4. *Entrepreneurial Performance Hypothesis*: Opportunities for knowledge-based entrepreneurship, and therefore performance of knowledge-based start-ups, is superior when they are able to access

[81] Audretsch, et al (2006), p. 49.
[82] ibid, pp. 49-51.

knowledge spillovers through geographic proximity to knowledge sources, such as universities, when compared to their counterparts without a close geographic proximity to a knowledge source.

5. *Entrepreneurial Access Hypothesis*: Knowledge-based entrepreneurial firms will strategically adjust the composition of their boards and managers toward higher levels of knowledge and human capital so they can contribute to the access and absorption of external knowledge spillovers.

6. *Entrepreneurial Finance Hypothesis*: Knowledge-based entrepreneurial firms will tend to be financed from equity-based sources, such as venture capital, and less typically from traditional debt-based sources, such as banks.

For readers with a background in calculus, the appendix to this section presents the formal derivation of these six hypotheses (the first two derivations require an understanding of calculus).

After introducing the six hypotheses in Chapter 4, Audretsch, et al (2006) report the results of their empirical tests of those hypotheses in the subsequent chapters of *Entrepreneurship and Economic Growth.* Their findings are an important part of the argument for linking an entrepreneurship program to economic development programs and policies presented in Volume I of this report.

APPENDIX TO SECTION VII: Derivation of Hypotheses Concerning the Determinants of Entrepreneurship and its Impact on Economic Performance[83]

Based on their arguments, Audretsch, et al (2006) derive six hypotheses concerning the determinants of entrepreneurship and its impact on economic growth[84]. This appendix presents the formal derivation of these hypotheses introduced in Section VII, above.

Derivation of the Endogenous Entrepreneurship Hypothesis
Equation VII-8 is the expression for the process of generating new opportunities. Investments in new knowledge are denoted by L_A in the model. Looking at the rate of change in the generation of new opportunities with respect to investments in new knowledge gives the following result:

$$\frac{d\dot{A}_{opp}}{dL_A} = (1-\theta)\cdot\delta\lambda L_A^{\lambda-1} A^{\varphi} \qquad \text{(VII-A.1)}$$

which is positive for all L_A and A^{φ}. Hence, opportunities increase with investments in new knowledge. Now, turning to the rate of change in the generation of new opportunities with respect to knowledge spillovers gives the following result:

$$\frac{d\dot{A}_{opp}}{dA^{\varphi}} = (1-\theta)\cdot\delta L_A^{\lambda} \qquad \text{(VII-A.2)}$$

which is positive for all L_A. Hence, opportunities increase with knowledge spillovers and therefore firms will locate near the source of spillovers, *ceteris paribus*, which suggests the next hypothesis: the Economic Performance Hypothesis.

[83] ibid, pp. 49-51.
[84] ibid, pp. 49-51.

Derivation of the Economic Performance Hypothesis
Based on the arguments given, Audretsch, et al (2006) restate the Romer production function given in Equation VII-1, as:

$$Q^{85} = K^{\alpha}(\theta r, A)^{(1-\alpha)} L_Q)^{(1-\alpha)} \tag{VII-A.3}$$

Where θr denotes the *realized permeability* of the knowledge filter. That is, that level of (1- θ) that has been taken on by start-up firms. Thus, θr ε [0, 1-θ] or θ ≤ θr ≤ 1. An increase in entrepreneurial activity increases the realized permeability of the knowledge filter (θr) and therefore the distance between the permeability of the knowledge filter (θ), and the *realized* permeability of the knowledge filter (θr). Deriving the rate of change in output with respect to the realized permeability of the knowledge filer yields:

$$\frac{dQ}{d\theta r} = (1-\alpha)\,\theta r^{-\alpha}\; K^{\alpha} A^{(1-\alpha)} L_Q^{(1-\alpha)} = \frac{1-\alpha}{\theta r} Q \tag{VII-A.4}$$

which is greater than 0 for all Q (= GDP = Y = Income). Thus, economic output increases with entrepreneurial activity.

Derivation of the Location Hypothesis
The third hypothesis that emerges from the Knowledge-Spillover Theory of Entrepreneurship addresses the location of entrepreneurial activity. The spatial dimension is critical here: though knowledge spills over, it is spatially bounded. Since a new-firm, start-up has been identified as a conduit for transmitting knowledge spillovers, and that knowledge spillovers are spatially bounded, it follows that knowledge-spillover based entrepreneurship is also spatially bounded. That is, local access is required to tap into the knowledge spillover that facilitates entrepreneurship based on technology transfer: hence, Audretsch, et al's (2006) formulation of the Location Hypothesis[86].

Derivation of the Entrepreneurial Performance Hypothesis
Research findings indicate that higher knowledge spillovers result in higher growth-rates for cities. Audretsch, et al (2006) argue that this relationship should also hold if the unit of observation is the knowledge firm[87]. Therefore, the entrepreneurial firm accessing knowledge spillovers should exhibit superior performance, hence, their formulation of the Entrepreneurial Performance Hypothesis.

Derivation of the Entrepreneurial Access Hypothesis
Knowledge spillovers are a necessary, but not sufficient, condition for firms to access and absorb external knowledge. Firms may also need to invest in absorptive capacity. Since entrepreneurial start-ups are usually constrained by size, such absorptive capacity, at least measured in absolute terms, is limited. This led to Audretsch, et al's (2006) formulation of the Entrepreneurial Access Hypothesis[88].

Derivation of the Entrepreneurial Finance Hypothesis
Even if entrepreneurial firms are able to access and absorb external knowledge spillovers, they will still most likely need an external source of financing. To address this challenge faced by many entrepreneurial start-ups, Audretsch, et al (2006) formulated the Entrepreneurial Finance Hypothesis[89].

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[85] Again, as noted above, Audretsch, et al's (2006) denote output as "Y". Here "Y" is replaced with "Q" for consistency.
[86] ibid, p. 50.
[87] ibid, p. 50.
[88] ibid, p. 50.
[89] ibid, p. 50.

VIII. SUMMARY AND CONCLUDING REMARKS

This volume detailed the formal framework that provides the context for the discussion on implementing the policies and programs in Volume I that addressed the issues and challenges identified in *Benchmarking Growth in Demand-Driven Labor Markets*. It elaborated on, and extended, the Audretsch, Keilbach, and Lehmann (2006) approach to the development of economic growth theory after World War II. They partition the Post-WW II era into three historical periods (1.) Technology as Exogenous: the Solow Model, (2.) Technology as Endogenous: the Romer-Lucas Model, and (3.) the Entrepreneurial Economy. Central to their approach, and that followed in Volume I, is the treatment of technology and knowledge by each of the three growth paradigms, and their implications about the changing role of the entrepreneur and the exploitation of knowledge for the development and introduction of new products and services into the market.

The presentation began with a brief introduction and review of the production function and some fundamental derived concepts and relationships. From Robert Solow's 1956 paper on, Neoclassical growth theory has been based on the production function. Thus, a basic grasp of this production relationship is essential for understanding the subsequent discussion of the evolution of economic growth theory since World War II, and was presented in this volume. But, before introducing Neoclassical growth theory, it was essential to introduce its antecedent, and the motivation for its development. The Harrod-Domar model was initially created to help analyze the business cycle; however, it was later adapted to explain economic growth. Its implications were that growth depends on the quantity of labor and capital, and that more investment leads to capital accumulation, which generates economic growth. The Harrod-Domar model predicts that if it is expected that output will grow, investment will increase to meet the extra demand. The problem arises when actual growth either exceeds or falls short of warranted growth expectations. A vicious cycle can be created where the difference is exaggerated by attempts to meet the actual demand, causing economic instability.

The first period of the Post-WW II Era can be characterized as the "Solow Economy." Robert Solow's 1956 article was largely addressed to the pessimism about the *razor's edge* path that the economy must maintain to sustain full-employment growth, which is built into the Harrod-Domar growth model. Solow's work changed the approach that economists took to study growth. From then on, the production-function model has been the basis for explaining the determinants of economic growth. The production-function approach relates measures representing two fundamental factors of production:

1. Physical Capital (K)

2. Unskilled Labor (L)

These two fundamental factor inputs were used as the basis for explaining variations in growth rates over time in a single country, or across countries in a cross-sectional context. The *unexplained residual*, which typically accounted for a large share of the unexplained variance in growth rates, was attributed to *technological change.* Solow acknowledged that technical change contributed to economic growth, but in terms of the formal model, it was considered "manna from heaven." First proposed by Romer (1986, 1987), *endogenous growth theory* maintained the orthodox Neoclassical growth-accounting framework, but dispensed with the need for an exogenous technology residual. Unlike Romer's focus on firm-specific capital, Lucas's version of the endogenous growth model is based on the *level of human capital*. According to the Lucas model, the portion of output attributed to the technology residual in the Neoclassical growth model, should actually be attributed entirely to labor through human capital acquisition. A fundamental implication emerging from the models of endogenous growth was that higher economic growth rates could be obtained through knowledge investments.

Critical to both the Romer and Lucas models is the internalizing of technological progress within the Neoclassical production-function framework by introducing knowledge as an explicit factor of production. In the Solow growth model, technology was exogenous (i.e., "manna from heaven") that resulted in an upward shift in the aggregate production function. In contrast, endogenous growth theory sought to identify the mechanism that explained technological progress over time, and to show that it was a product of the internal processes of the economy.

These models, whether Neoclassical, and its extension to Endogenous Growth, or Keynesian/Post-Keynesian, were originally developed to explain the growth process at the national level. Subsequently, attempts were made to adapt these models to explain economic growth at the regional level. In the last half of the 20th Century, both the Harrod-Domar and Neoclassical models were adapted to explain regional growth. In 1964, Borts and Stien first adapted the one-sector Neoclassical model to explain regional growth. In response to empirical evidence that contradicted the predictions of the one-sector model, Borts and Stien (1968) then developed the two-sector Neoclassical model to explain growth at the regional level. In 1969, Richardson developed a regional version of the Harrod-Domer model. While Neoclassical growth theory can be thought of as focusing on problems of supply, assuming sufficient demand, Keynesian-based growth theories can be thought of as focusing on deficiencies in demand that constrain production (output) and growth. The Harrod-Domar model, a Keynesian-oriented model, focuses on the interaction between supply *and* demand, and how the interplay and feedbacks between the two drive growth and fluctuations in the economy. Another set of Keynesian-based growth and development models, circular-cumulative causation, developed by Myrdal and formalized by Kaldor, also focuses on the interaction between supply and demand, and how this interaction ignites a chain-reaction process that generates a virtuous circle of cumulative causation propelling a region toward a trajectory of growth and development.

In their theory of endogenous entrepreneurship, Audretsch, Keilbach, and Lehmann (2006) sought to go beyond the limits of endogenous growth theory and emphasize the critical delineation between *information* and *knowledge*. While advances in information technology have rendered the cost of transmitting information across space trivial, the cost of transferring knowledge across space still follows a steep decay function. In addition to the distinction between information and knowledge, also critical to the development of a model of entrepreneurship and growth, is the idea that there is a barrier to translating new knowledge into new *economic* knowledge. Audretsch, et al (2006) formalize this idea in their concept of the *knowledge filter*. Further, not only is the knowledge filter the consequence of the basic conditions inherent in new knowledge but, it is also what creates the opportunity for entrepreneurship in the *knowledge spillover theory of entrepreneurship.*

Their observation that knowledge conditions dictate the relative advantages in exploiting opportunities arising from investments in knowledge of incumbents versus small and large enterprises is predicated on the distinction between two knowledge regimes: the *routinized technological regime* versus the *entrepreneurial technological regime*. The routinized technological regime reflects knowledge conditions where the large incumbent firms have the innovative advantage. Conversely, in the entrepreneurial technological regime, knowledge conditions give the advantage to small firms. In their formal model of endogenous entrepreneurship, Audretsch, et al (2006) emphasize, not only that the capacity of each regional economy to generate entrepreneurial spillovers and commercialize knowledge is not the same, but, in addition, just as the knowledge filter should not be assumed to be impermeable, the capacity of a region's economy to generate knowledge spillovers via entrepreneurship to permeate the knowledge filter should not be assumed to be automatic. Consequently, the remaining untapped part represents opportunities that can be taken on by new firms. They denote this part as *entrepreneurial opportunities*, and it is explicitly expressed as a term in their specification of the production function.

Based on the above model of the knowledge spillover theory of entrepreneurship and economic growth, as well as their framework for analyzing the recognition of and then acting upon entrepreneurial opportunities, Audretsch, et al derive the following hypotheses:

- *Endogenous Entrepreneurship Hypothesis*: Entrepreneurship will be greater in the presence of higher investments in new knowledge, ceteris paribus.

- *Economic Performance Hypothesis*: Entrepreneurial activity will increase the level of economic output since entrepreneurship serves as a mechanism facilitating the spillover and commercialization of knowledge.

- *Location Hypothesis:* Knowledge spillover entrepreneurship will tend to be spatially located within close geographic proximity to the source of knowledge actually producing that knowledge.

- *Entrepreneurial Performance Hypothesis*: Opportunities for knowledge-based entrepreneurship, and therefore performance of knowledge-based start-ups, is superior when they are able to access knowledge spillovers through geographic proximity to knowledge sources, such as universities, when compared to their counterparts without a close geographic proximity to a knowledge source.

- *Entrepreneurial Access Hypothesis*: Knowledge-based entrepreneurial firms will strategically adjust the composition of their boards and managers toward higher levels of knowledge and human capital so they can contribute to the access and absorption of external knowledge spillovers.

- *Entrepreneurial Finance Hypothesis*: Knowledge-based entrepreneurial firms will tend to be financed from equity-based sources, such as venture capital, and less typically from traditional debt-based sources, such as banks.

After introducing their hypotheses in Chapter 4, Audretsch, et al (2006) report the results of their empirical tests of those hypotheses in the subsequent chapters of *Entrepreneurship and Economic Growth.* Their findings are an important part of the argument for linking an entrepreneurship program to economic development programs and policies presented in this report.

This volume (Volume II) has been directed toward those readers who were interested in pursuing a more in-depth discussion of the economic theories guiding the development of the arguments made in Volume I. To that end, it is hoped that this volume has provided a more detailed and technical development of the presentation in Section III of Volume I, A FORMAL CONTEXT FOR A GROWTH AND DEVELOPMENT STRATEGY. In addition, the references should provide some resources for further investigation into this, and related, areas concerning the regional growth and development process.

IX. REFERENCES

Audretsch, David B., Max C. Keilbach, and Erik E. Lehmann, ENTREPRENEURSHIP AND ECONOMIC GROWTH (2006) Oxford University Press: New York.

Borts, George H. and Jerome L. Stien, ECONOMIC GROWTH IN A FREE MARKET (1964) Columbia University: New York

---------------------------- *Regional Growth and Maturity in the United States: A Study of Regional Structural Change*, in L. Needleman Ed., REGIONAL ANALYSIS (1964) Penguin: Hammondsworth, pp. 159-97, Original Edition in SCHWEIZERISCHE ZEITSCHRIFT für VOLKSWIRTSCHAFT und STATISTIK (1962) (98): 290-321.

Call, Steven T. and William L. Holahan, MICROECONOMICS, 2nd Ed. (1983) Wadsworth Publishing: Belmont, CA. Ch. 5.

Cannan, Edwin, *The Origin of the Law of Diminishing Returns*, 1813-15, ECONOMIC JOURNAL (1892): 2

Capello, Roberta, REGIONAL ECONOMICS (2007) Routeledge: New York

Chiang, Alpha, FUNDAMENTAL METHODS OF MATHEMATICAL ECONOMICS, 3rd Edition (1984) McGraw-Hill: New York.

Cortright, Joseph, *New Growth Theory, Technology, and Learning* (2001) U.S. Economic Development Administration: Washington, p. 5.

Denzin, N. SOCIOLOGICAL METHODS: A Sourcebook (1978) NY: McGraw Hill, 2nd ed.

Domar, D. (1946), Capital Expansion, Rate of Growth and Employment, ECONOMETRICA, Vol. 14.

Easterly, William, *The Ghost of Financing Gap* (July 1997) DRAFT.

Ghali, Mohab, M. Akiyama, and J. Fujiwara, *Factor Mobility and Regional Growth*, REVIEW OF ECONOMICS AND STATISTICS (60): 78-84

Harrod, R. F. (1939), *An Essay in Dynamic Theory*, ECONOMIC JOURNAL, Vol. 49, No. 1

Hoyt, H. and A. Wiemer, PRINCIPLES OF URBAN REAL ESTATE (1939) Ronald Press: New York

Jones, Charles I., INTRODUCTION TO ECONOMIC GROWTH (2002) 2nd Ed. W.W. Norton: New York

Kaldor, N., *The Case of Regional Policies*, SCOTISH JOURNAL OF POLITICAL ECONOMY (1970) (3): 337-348.

Keynes, J.M., THE GENERAL THEORY OF EMPLOYMENT MONEY AND INTEREST (1936)

Lucas, R. E., *On the Mechanics of Economic Development*, J. OF MONETARY ECONOMICS (1988) (22): 3-42

Malizia, Emil E. and Edward J. Feser, UNDERSTANDING LOCAL ECONOMIC DEVELOPMENT (1999, 2000) Center for Urban Policy Research: New Brunswick, N.J.

Mansfield, Edwin, MICROECONOMICS: Theory and Applications, 2nd Ed. (1975) W.W. Norton: New York, Ch. 5.

McCann, Philip, URBAN AND REGIONAL ECONOMICS (2001) Oxford University Press: New York.

McDonald, John F., FUNDAMENTALS OF URBAN ECONOMICS (1997) Prentice Hall: Upper Saddle River, N.J.

Musgrave, Richard A and Peggy Musgrave, PUBLIC FINANCE IN THEORY AND PRACTICE (1959) McGraw-Hill, New York.

Myrdal, G. ECONOMIC THEORY OF UNDER-DEVELOPED REGIONS (1957) Duckworth: London

Reynolds, R. Larry, Production and Cost, BASIC MICROECONOMICS (2000)

Richardson, Harry, W., REGIONAL ECONOMICS (1969) World University, Redwood Press: Trowbridge, Wiltshire

Romer, P. M., *Growth Based on Increasing Returns Due to Specialization*, AMERICAN ECONOMIC REVIEW (1987) (77.2): 56-62

Romer, P. M., *Increasing Returns and Long-Run Growth*, J. OF POLITICAL ECONOMY (1986) (94): 1002-37

Schumpeter, Joseph A., CAPITALISM, SOCIALISM, AND DEMOCRACY (1936)

Solow, R.M. (1956) A Contribution to the Theory of Economic Growth, QUARTERLY JOURNAL OF ECONOMICS. Vol. 70 (1) pp. 65-94.

Solow, R.M. (1987) GROWTH THEORY: An Exposition, Oxford University Press: New York

Swan, Trever, *Economic Growth and Capital Accumulation*, THE ECONOMIC RECORD (1956) 32 (November): 334-361.

Tiebout, C., THE COMMUNITY ECONOMIC BASE STUDY (1962) Committee for Economic Development: New York

Uwasu, Michiniori, *The Solow Growth Model* (Spring 2006) APEC 3006

Valdẽs, Benigno, ECONOMIC GROWTH: Theory, Empirics, and Policy (1999) Edward Elgar: Northhampton, MA.

Verdoorn, P.J., *Fattori che Regolano lo Sviluppo della Prodduttivita del Lavoro*, L'INDUSTRIA (1949) (1): 3-10, Translated by A.P. Thirlwall and D. Ironmonger in J. Perkins and T. Hoa (Eds) NATIONAL INCOME AND ECONOMIC PROGRESS: Essays in Honor of Colin Clark (1988), Macmillen: London